Publications

1. *The Gourava Index of Four Operations on Graphs*, International Journal of Mathematical Combinatorics, 4(1), 2018, $65 - 76$, ISSN:-1937-1055.

2. *Computation of Adriatic indices of certain operators of regular and complete Bipartite graphs*, Advanced studies in contemporary Mathematics, 28(2), 2018, $231-244$, ISSN: 12293067, (**UGC Journal No.:11665**)(Scopus).

3. *Adriatic Indices and Sanskruti index envisage of Carbon Nanocone*, TWMS J. App. Eng. Math., 9(4), 2019, $830-837$, ISSN:21461147 (Scopus),(**UGC No:48837**).

4. *Some Adriatic indices of Dutchwind mill graph using graph operators*, Advances and applications in Discrete Mathematics, 22(2), 2019, $127-137$, ISSN: 09741658,(WOS)(ESCI).

5. *The Degree Sequences of S-Corona graphs*, International Journal of Advance and Innovative Research, 6(2), 2019, 191− 196, ISSN : 23947780, (**UGC No:63571**).

6. *Some Topological indices computing results and subdivision of L(4,82)*, International Journal of Advance and Innovative Research, 6(2), 2019, 197− 200, ISSN : 23947780, (**UGC No:63571**).

7. *S-Corona operations of standard graphs in terms of Degree Sequences*, Proceeding of the jangjeon Mathematical Society, 2019(Accepted).

8. *Investigation on splice graphs by exploting certain topological indices*, Proceeding of the jangjeon Mathematical Society, 2019(Accepted).

9. *Operations Of Triglyceride via Adriatic Indices*, (communicated).

Presentations

Papers presented in International/National Conferences

1. Attended a Pre-conference workshop on recent advances in signed Graphs and their applications at Siddaganga institute of Technology, Tumakuru, held on $06^{th} - 08^{th}$ June 2016.

2. Attended an International conference on Discrete Mathematics 2016 [ICDM-2016] and graph theory day XII at Siddaganga institute of Technology, Tumakuru, held on $09^{th} - 11^{th}$ June 2016.

3. Presented a paper on Operations on Dutch windmill graph via Adriatic indices in National conference on Geometry, Topology and their Applications at Karnataka University, Dharwad, held on $03^{rd} - 04^{th}$ August 2016.

4. Attended an Open lecturer series on recent advances in Science and Technology: Mathematica at MES Degree college, Bengaluru, held on 19^{th} September 2016.

5. Attended a National workshop on Innovative research techniques at Central University of Karnataka, Kalaburagi, held on $25^{th} - 26^{th}$ November 2016.

6. Presented a poster on Scientific operations of Topological indices in 9^{th} Karnataka Science and Technology Academy annual conference on Science, Technology and innovations in the 21^{st} century at Christ University, Bengaluru, held on $20^{th} - 21^{st}$ December 2016.

7. Presented a poster on Certain Adriatic indices envisage of carbon nanocone in Karnataka Science and Technology Academy National conference on Impact of Science and technology on society and economy at VSK University, Ballari, held on $08^{th} - 09^{th}$ March 2017.

8. Attended a Summer school 2017 on Social Networks workshop at IIT Ropar, Punjab, held on 28^{th} May to 02^{nd} June 2017.

9. Presented a paper on Operations of Triglyceride via Adriatic indices in 32^{nd} annual conference of Ramanujan Mathematical Society at Rani Channamma University, Belagavi, held on $23^{rd} - 25^{th}$

June 2017.

10. Attended a National conference on Science and Technology education at University of Agricultural Science, Raichur, held on $21^{st} - 22^{nd}$ July 2017.

11. Attended a 1^{st} International conference on Colloborative Research in Mathematical Sciences ICCRMS17 at KG College of arts and Science, Coimbatore, held on 23^{rd} September 2017.

12. Attended a Post graduate special lecturer series in Mathematics organized by Karnataka Science and Technology Academy and Vijayanagara Sri Krishnadevaraya University, Ballari, held on $09^{th} - 10^{th}$ November 2017.

13. Presented a paper on Molecular descriptors of carbon nanocone of some graph operators in National conference on Recent advances in Mathematical Sciences and applications at Tumkur University, Tumkur, held on $01^{st} - 02^{nd}$ December 2017.

14. Attended a workshop on Role of Stastistics in Scientific Research at Karnataka Science and Technology Academy, Bengaluru, held on $04^{th} - 05^{th}$ January 2018.

15. Presented a poster on Computation of Adriatic indices of certain operators of regular and complete bipartite graphs in Karnataka Science and Technology Academy 10^{th} annual national conference at Reva University, Bengaluru, held on $18^{th} - 19^{th}$ January 2018.

16. Attended a workshop on Algebraic Graph Theory at NITK, Surathkal, held on $25^{th} - 30^{th}$ January 2018.

17. Presented a paper on Bounds for the topological indices of jump graph in International conference on advances in pure and applied Mathematics at school of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, held on $06^{th} - 08^{th}$ September 2018.

18. Presented a paper on Inventive on topological indices and M-polynomials of polycyclic aromatic Hydrocarbons in International conference on recent trends in Mathematics at Raja lakhamagouda Science institute, Belagavi, held on $27^{th} - 29^{th}$ December 2018.

19. Presented a poster on Investigation on new operations related to the R-corona of graphs via F-Index in Karnataka Science and

Technology Academy 11^{th} annual national conference on New vistas in Science and technology for common good at NMKRV College for Women, Bengaluru, held on $01^{st} - 02^{nd}$ February 2019.

20. Presented a paper on F-Index of graphs based on new operations related to the corona of graphs in International conference on emerging trends in graph theory (ICETGT-2019) at Christ [Deemed to be University], Bengaluru, held on $27^{th} - 28^{th}$ February 2019.

21. Presented a paper on A comparative investigation of molecular weight of some Carbohydrates and topological indices in National Symposium on Mathematics and its applications [NSMA] at Bangalore University, Bengaluru, held on 27^{th} April 2019.

22. Presented a paper on The Degree sequences of S-Corona Graphs in 2^{nd} International conference on Global Advancement of Mathematics-2019 at Acharya Institute of Graduate Studies, Bengaluru, held on $25^{th} - 26^{th}$ June-2019.

23. Presented a paper on S-Corona operations of standard graphs in terms of Degree Sequences in International conference on Number

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Abstract

This research work primarily frame-up with some topological indices on the different types of general graphs, molecular graphs and graph operations. Analyzed some explicit expression for the Gourava index of four operation on graphs in terms of first and second Zagreb index. The investigation on generalized version of some adriatic indices of Dutch windmill graph using graph operators such as subdivision, line and derived graphs. We frame-up with the general expression for some discrete adriatic indices and Sanskruti index of carbon nanocones $CNC_m[n]$. *The computation of certain degree based adriatic indices of triglyceride using different graph operators. The explicit interpretation of inverse sum indeg, reformulated Zagreb, atom-bond connectivity and Shegehalli and Kanabur*1 *indices in terms of the graph size and maximum or minimum vertex degrees of special splice graphs are obtained. Determine the DS of S-vertex(edge) corona and S-edge neighbourhood corona operations of standard graphs. Also, generalizing DS of S-vertex(edge) corona and S-vertex(edge) neighbourhood corona operations of graphs.*

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Chapter 1

Prelude

1.1 Brief History

A topological index or a connectivity index is a sort of atomic descriptor or molecular descriptor that is calculated based on the atomic graph or molecular system of a chemical compound. Topological indices are the numerical parameter of a graph that describe its topology and is normally graph invariant. Atomic graph of topological indices are formed on the basis of shifting into a number which characterize the graph topology [39]. Significance of topological index started by a chemist Harold Wiener in the year 1947 [2] developed the most widely known topological descriptor, the Wiener index, and used it to determine the physical properties of types of alkanes known as paraffin.

Based on the information given by the International Academy of

Mathematical Chemistry ($IAMC$) [28]. Vukicevic and Gasperov introduced 148 bond-additive Discrete Adriatic indices and shown highly correlated with physical properties in chemical science and there was tremendous research identified with topological records and their properties.

A graph consists of vertices and edges whereas the atomic graph represents atoms and bonds. A topological index can be computed from the atomic graph and used to characterize some properties of the underlying molecule. [12] These calculated numerical values of topological indices are used in the development of studies in the "Quantitative Structure-Property Relationships ($QSPR$) and in Quantitative Structure-Activity Relationships ($QSAR$)".

In this research work, we consider a certain topological indices that are proved to be effective. In appropriate, topological indices such as first and second zagreb index, Gourava index, Inverse sum index, Misbalance index and few discrete adriatic indices are preferred for the research. Moreover, we enclose our work to degree based topological indices on different classes of graphs and graph operations such as F-sum graphs, Derived graph, S-vertex(edge) splice graph, S-vertex(edge) neighbourhood splice graph, $S(G)$, $L(G)$, $R(G)$, $Q(G)$, $T(G)$ respectively.

1.2 Essential of topological indices

The following table contains definitions are utilized for the forthcoming chapters [1, 10, 19, 24, 27, 34, 38, 42, 49, 50].

Name of index	Representation
ISI	$\sum_{uv \in E(G)} \frac{d_u d_v}{d_u + d_v}$
M_1	$\sum_{uv \in E(G)} [d_u + d_v]$
AZI	$\sum_{uv \in E(G)} \left[\frac{d_u d_v}{d_u + d_v - 2} \right]^3$
M_2	$\sum_{uv \in E(G)} [d_u d_v]$
SDD	$\sum_{uv \in E(G)} \frac{d_u^2 + d_v^2}{d_u d_v}$
GO_1	$\sum_{uv \in E(G)} [d_u + d_v + d_u d_v]$
LM_1	$\sum_{uv \in E(G)} 2 \left[\frac{lnd_u}{d_u} + \frac{lnd_v}{d_u} \right]$
$\overline{LM_1}$	$\sum_{uv \in E(G)} ln \left[d_u + d_v \right]$
MLD	$\sum_{uv \in E(G)} \mid lnd_u - lnd_v \mid$
MRD	$\sum_{uv \in E(G)} \mid \sqrt{d_u} - \sqrt{d_v} \mid$
MHD	$\sum_{uv \in E(G)} \mid 2^{-d_u} - 2^{-d_v} \mid$
$MIRD$	$\sum_{uv \in E(G)} \mid \frac{1}{\sqrt{d_u}} - \frac{1}{\sqrt{d_v}} \mid$
MD	$\sum_{uv \in E(G)} \mid d_u - d_v \mid$
$MLSD$	$\sum_{uv \in E(G)} \mid ln^2 d_u - ln^2 d_v \mid$
$ISLSD$	$\sum_{uv \in E(G)} \left[\frac{1}{\sqrt{lnd_u} + \sqrt{lnd_v}} \right]$
SK_1	$\sum_{uv \in E(G)} \left[\frac{d_u + d_v}{2} \right]$

EM_1	$\sum_{uv\in E(G)} d(e)^2$
ABC	$\sum_{uv\in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}}$
F	$\sum_{u\in V(G)} d_u^3$
H	$\sum_{uv\in E(G)} \left[\frac{2}{d_u+d_v}\right]$
$\mathcal{S}$	$\sum\limits_{uv\in E(G)} \left[\frac{S_u S_v}{S_u+S_v-2}\right]^3$

1.3 Basic terminologies

A non-empty vertex set $V = V(G)$ of a graph named as vertices with a disordered pairs of different points of edge set $E = E(G)$ of G. The order of vertex V and size E is represented as (n, m).

Path is a finite or infinite walk and no vertex is repeated, a closed path is called cycle and complete graph with n-vertices having each vertex degree as $(n - 1)$.

A graph $G = (V = \{V_1, V_2\}, E)$ interfaces every vertex from set V_1 to each vertex from set V_2 [33] is called a complete bipartite graph . If a solitary vertex belongs to one set and all other vertices belong to another set in a complete bipartite graph is known as a Star graph . A graph having each vertex degree is r and is called r-regular graph.

Definition 1.3.1. The **Graph Distance** [21] is the minimal path connecting between distance $d(u, v)$ of any two vertices u and v in G.

Definition 1.3.2. The **Total graph** $T(G)$ [14,29] is a non-empty vertex set $V(G) \cup E(G)$ in $T(G)$ and any two vertices of $T(G)$ are said to be adjacent when they are either incident or adjacent in G.

Definition 1.3.3. The **Derived graph** [4,25,46] of G, symbolized by $G^{\dagger}$ is the graph having set $V(G)$, in which their length in G is two in case the two vertices are adjacent in $G^{\dagger}$.

Definition 1.3.4. The **Subdivision graph** $S(G)$ [7,31,35] of a graph G is the graph obtained by adding a new vertex of degree 2 in each edge of G.

Definition 1.3.5. The **Line graph** $L(G)$ [23,44] of a simple graph G is the graph in which there is a one to one correspondence between vertices of $L(G)$ and edges of G and two vertices of $L(G)$ are connected by an edge if and only if the corresponding edges are adjacent in G.

Definition 1.3.6. The $Q(G)$ **or semi-total line graph** [6,17] $T_1(G)$ is the graph having $V(G) \bigcup E(G)$ where two vertices of $T_1(G)$ are adjacent if and only if

(i) one is a vertex of G and the other is an edge of G incident to that vertex or **(ii)** they are adjacent edges of G.

Definition 1.3.7. The $R(G)$ **or semi-total point graph** [32] $T_2(G)$ of G is the graph having $V(G)\bigcup E(G)$ where two vertices of $T_2(G)$ are adjacent if and only if

(i) one is a vertex of G and the other is an edge of G incident with it or **(ii)** they are adjacent vertex of G.

Definition 1.3.8. The **Cartesian product** is an important method to construct a ample graph and play vital role in the design and analysis the network [18]. The cartesian product of two connected graphs G and H, which is denoted by $G\Box H$, is a graph such that the set of vertices is $V(G)\Box V(H)$ and two vertices (p_1, q_1) and (p_2, q_2) of $G\Box H$ are adjacent if and only if $p_1 = p_2$ and q_1 is adjacent with q_2 in H otherwise $q_1 = q_2$ and p_1 is adjacent with p_2 in G.

1.4 Summary of the thesis

The primary objective of this thesis is to focus on analyzing the distinct types of topological indices of graph operations. In chapter 2, expressions for the Gourava index of four operation on graphs in terms of first and second zagreb indices. Chapter 3 deals with investigation of adriatic indices for the Dutch windmill graph of graph operators. Chapter 4 is concentrated on general expression for some discrete adriatic indices and Sanskruti index of carbon nanocones $CNC_m[n]$.

In chapter 5 computed the degree based adriatic indices of graph operators of triglyceride. Chapter 6 inverstigation of lower and upper bounds on splice graphs through topological indices. In [47] Tyshkevich et. al., established a correspondence between DSs of graph. Inspired from that in chapter 7 obtain the DSs of S-corona operations of standard graphs. Chapter 8 generalization of S-corona operators of different graphs.

Finally, conclusion and future scope on topological indices are tinted. The bibliography is placed at the end and appropriate references are cited throughout the thesis.

Chapter 2

The Gourava index of four operations on graphs

2.1 Preliminaries

Let G and H be two connected graphs. M. Eliasi, B. Taeri [15] introduced four new operations named as F-sum graphs, on these graphs that are based on S, T_2, T_1, T as follows.

Let F be one of the symbols S, T_2, T_1 or T [5,11,43]. The F-sum denoted by $G +_F H$ of graphs G and H, is a graph with the set of vertices

$V(G+_F H) = (V(G) \bigcup E(G)) \times V(H)$ and (p_1, p_2) $(q_1, q_2) \in E(G+_F H)$, if and only if $p_1 = p_2 \in V(G)$ and $q_1q_2 \in E(H)$ or $q_1 = q_2$ and $(p_1, p_2) \in E(F(G))$.

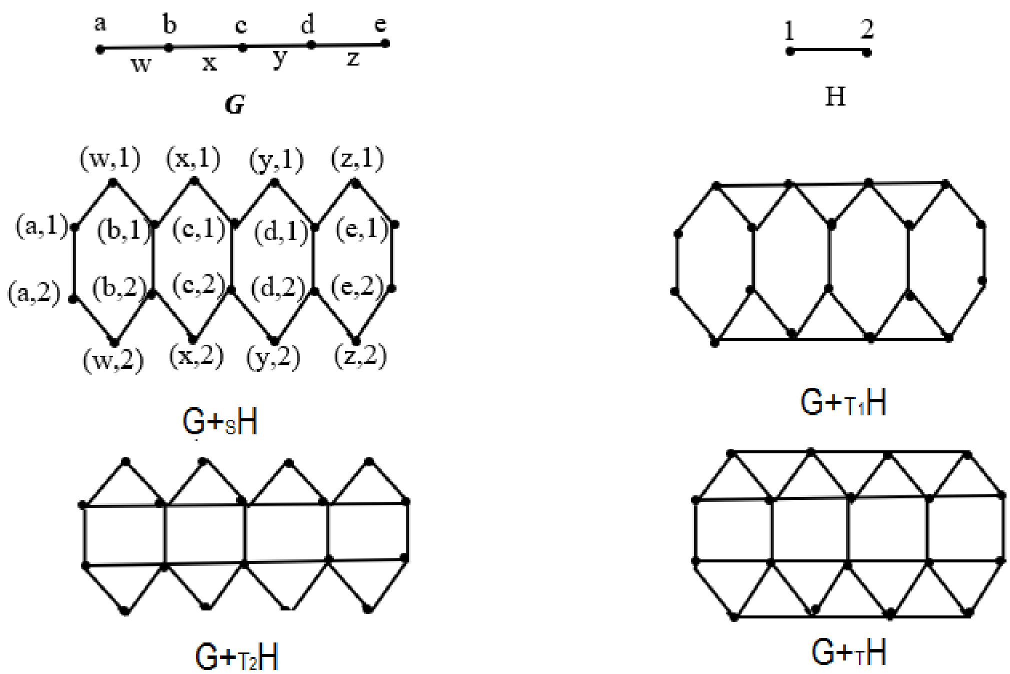

Figure 1: Graph G, H and G+$_F H$.

In this chapter, we discuss main results of Gourava index of F-sum of graphs.

2.2 Relation connecting topological indices of Gourava index of F-sum in terms of Gourava, first and second zagreb indices

Theorem 1. *Let G and H be two connected graphs. Then*

$$GO_1(G +_s H) = n_H GO_1(G) + n_G GO_1(H) + e_H M_1(G) + 2e_G M_1(H) + 8n_H e_G + 12e_H e_G.$$

Proof. From the definition of Gourava index,

$$\begin{aligned}
GO_1(G +_s H) &= \sum_{(p_1,q_1)(p_2,q_2)\in E(G+_sH)} d_{G+_sH}(p_1,q_1) + d_{G+_sH}(p_2,q_2) \\
&+ d_{G+_sH}(p_1,q_1) d_{G+_sH}(p_2,q_2) \\
&= \sum_{p_1\in V(G)} \sum_{q_1q_2\in E(H)} d_{G+_sH}(p_1,q_1) + d_{G+_sH}(p_1,q_2) \\
&+ d_{G+_sH}(p_1,q_1) d_{G+_sH}(p_1,q_2) \\
&+ \sum_{q_1\in V(H)} \sum_{p_1p_2\in E(S(G))} d_{G+_sH}(p_1,q_1) + d_{G+_sH}(p_1,q_2) \\
&+ d_{G+_sH}(p_1,q_1) d_{G+_sH}(p_1,q_2)
\end{aligned}$$

$$= I_1 + I_2. \qquad (1)$$

Where I_1, I_2 are the sums of the above terms, in order.

$\forall$ vertex $p_1 \in V(G)$ and $q_1q_2 \in E(H)$ we get

$$\begin{aligned}
I_1 &= \sum_{p_1\in V(G)} \sum_{q_1q_2\in E(H)} d_G(p_1) + d_H(q_1) + d_G(p_1) + d_H(q_2) \\
&+ [d_G(p_1) + d_H(q_1)][d_G(p_1) + d_H(q_2)] \\
&= \sum_{p_1\in V(G)} \sum_{q_1q_2\in E(H)} 2d_G(p_1) + d_H(q_1) + d_H(q_2) + d_G^2(p_1) \\
&+ d_G(p_1)[d_H(q_1) + d_H(q_2)]d_H(q_1)d_H(q_2) \\
&= \sum_{p_1\in V(G)} 2e_Hd_G(p_1) + M_1(H) + e_Hd_G^2(p_1) + d_G(p_1)M_1(H) + M_2(H) \\
&= 4e_He_G + n_GGO_1(H) + e_HM_1(G) + 2e_GM_1(H).
\end{aligned}$$

$\forall$ edge $p_1p_2 \in E(S(G))$, where the vertex $p_1 \in V(G)$, $p_2 \in V(S(G)) - V(G)$ and $q_1 \in V(H)$, since $|E(S(G))| = 2|E(G)|$.

$$\begin{aligned}
I_2 &= \sum_{q_1\in V(H)} \sum_{p_1p_2\in E(S(G))} d_{S(G)}(p_1) + d_H(q_1) + d_{S(G)}(p_2) \\
&+ [d_{S(G)}(p_1) + d_H(q_1)]d_{S(G)}(p_2) \\
&= \sum_{q_1\in V(H)} GO_1(S(G)) + 2e_Gd_H(q_1)) + 2e_Gd_H(q_1)) \\
&= n_HGO_1(S(G)) + 8e_He_G.
\end{aligned}$$

We know that, $M_1[S(G)] = M_1(G) + 4e_G$ and $M_2[S(G)] = M_2(G) + 4e_G$ therefore $GO_1(S(G)) = GO_1(G) + 8e_G$

$$I_2 = n_H GO_1(G) + 8n_H e_G + 8e_H e_G.$$

Substituting I_1 and I_2 in (1) we get required result.

$$\begin{aligned} GO_1(G +_s H) &= n_H GO_1(G) + n_G GO_1(H) + e_H M_1(G) + 2e_G M_1(H) \\ &+ 8n_H e_G + 12e_H e_G. \end{aligned}$$

□

Theorem 2. *Let G and H be two connected graphs. Then*

$$\begin{aligned} GO_1(G +_{T_1} H) &= n_G GO_1(H) + 5e_H M_1(G) + 3e_G M_1(H) + 2n_H M_1(G) \\ &+ 2e_G n_H M_1(G) + 10e_H e_G + n_H \sum_{\substack{u_iu_j \in E(G), \\ u_ju_k \in E(G)}} d_G(u_i)[1 + d_G(u_k)] \\ &+ d_G(u_k)[1 + d_G(u_j)] + d_G(u_j)[d_G(u_i) + d_G(u_j)]. \end{aligned}$$

Proof. consider,

$$GO_1(G +_{T_1} H) = \sum_{(p_1,q_1)(p_2,q_2) \in E(G +_{T_1} H)} d_{G+_{T_1}H}(p_1, q_1) + d_{G+_{T_1}H}(p_2, q_2)$$

$$
\begin{aligned}
&+ \; d_{G+_{T_1}H}(p_1,q_1)d_{G+_{T_1}H}(p_2,q_2)\\
&= \sum_{p_1\in V(G)}\sum_{q_1q_2\in E(H)} d_{G+_{T_1}H}(p_1,q_1)+d_{G+_{T_1}H}(p_1,q_2)\\
&+ \; d_{G+_{T_1}H}(p_1,q_1)d_{G+_{T_1}H}(p_1,q_2)\\
&+ \sum_{q_1\in V(H)}\sum_{p_1p_2\in E(T_1(G))} d_{G+_{T_1}H}(p_1,q_1)+d_{G+_{T_1}H}(p_2,q_1)\\
&+ \; d_{G+_{T_1}H}(p_1,q_1)d_{G+_{T_1}H}(p_2,q_1).
\end{aligned}
$$

The edge set $E(T_1(G))$ split in to $E(S(G))$ and $E(L(G))$.

Let $E(T_1(G))=\alpha_1$, $V(G)=\beta$, $V(T_1(G))-V(G)=\gamma_1$

$$
\begin{aligned}
GO_1(G+_{T_1}H) &= \sum_{p_1\in V(G)}\sum_{q_1q_2\in E(H)} d_{G+_{T_1}H}(p_1,q_1)+d_{G+_{T_1}H}(p_1,q_2)\\
&+ \; d_{G+_{T_1}H}(p_1,q_1)d_{G+_{T_1}H}(p_1,q_2)+\sum_{q_1\in V(H)}\sum_{\substack{p_1p_2\in\alpha_1,\\ p_1\in\beta,\\ p_2\in\gamma_1}} d_{G+_{T_1}H}(p_1,q_1)\\
&+ \; d_{G+_{T_1}H}(p_2,q_1)+d_{G+_{T_1}H}(p_1,q_1)d_{G+_{T_1}H}(p_2,q_1)\\
&+ \sum_{q_1\in V(H)}\sum_{\substack{p_1p_2\in\alpha_1,\\ p_1,p_2\in\gamma_1}} d_{G+_{T_1}H}(p_1,q_1)+d_{G+_{T_1}H}(p_2,q_1)\\
&+ \; d_{G+_{T_1}H}(p_1,q_1)d_{G+_{T_1}H}(p_2,q_1)\\
&= J_1+J_2+J_3. \qquad (2)
\end{aligned}
$$

Where J_1, J_2, J_3 are the sums of the above terms, in order

$$\begin{aligned}
J_1 &= \sum_{p_1\in V(G)}\sum_{q_1q_2\in E(H)} 2d_{T_1(G)}(p_1)+d_H(q_1)+d_H(q_2)\\
&+ [d_{T_1(G)}(p_1)+d_H(q_1)][d_{T_1(G)}(p_1)+d_H(q_2)]\\
&= \sum_{p_1\in V(G)}\sum_{q_1q_2\in E(H)} 2d_{T_1(G)}(p_1)+d_H(q_1)+d_H(q_2)+d^2_{T_1(G)}(p_1)+d_{T_1(G)}(p_1)d_H(q_2)\\
&+ d_H(q_1)d_H(q_2)+d_{T_1(G)}(p_1)d_H(q_1)\\
&= \sum_{p_1\in V(G)} 2e_Hd_G(p_1)+GO_1(H)+e_Hd^2_G(p_1)+d_G(p_1)d_H(q_2)+d_G(p_1)d_H(q_1)\\
&= n_GGO_1(H)+e_HM_1(G)+e_GM_1(H)+2e_He_G.
\end{aligned}$$

$$\begin{aligned}
J_2 &= \sum_{q_1\in V(H)}\sum_{\substack{p_1p_2\in\alpha_1,\\ p_1\in\beta,\\ p_2\in\gamma_1}} [d_{T_1(G)}(p_1)+2d_H(q_1)+d_{T_1(G)}(p_2)]\\
&+ d_{T_1(G)}(p_1)+d_H(q_1)][d_{T_1(G)}(p_2)+d_H(q_1)]\\
&= \sum_{q_1\in V(H)}\sum_{\substack{p_1p_2\in\alpha_1,\\ p_1\in\beta,\\ p_2\in\gamma_1}} [d_G(p_1)+2d_H(q_1)+d_{T_1(G)}(p_2)]\\
&+ [d_G(p_1)+d_H(q_1)][d_{T_1(G)}(p_2)+d_H(q_1)]\\
&= \sum_{q_1\in V(H)}\sum_{\substack{p_1p_2\in\alpha_1,\\ p_1\in\beta,\\ p_2\in\gamma_1}} d_G(p_1)+2d_H(q_1)+d_{T_1(G)}(p_2)+d_G(p_1)d_{T_1(G)}(p_2)\\
&+ d_G(p_1)d_H(q_1)+d_H(q_1)d_{T_1(G)}(p_2)+d^2_H(q_1)
\end{aligned}$$

$$= \sum_{q_1 \in V(H)} \sum_{p_1 \in V(G)} d_G(p_1)[d_G(p_1) + 2d_H(q_1) + d_G(p_1)d_H(q_1) + d_H^2(q_1)]$$

$$+ \sum_{q_1 \in V(H)} \sum_{\substack{p_1p_2 \in \alpha_1, \\ p_1 \in \beta, \\ p_2 \in \gamma_1}} d_{T_1(G)}(p_2) + d_G(p_1)d_{T_1(G)}(p_2) + d_H(q_1)d_{T_1(G)}(p_2).$$

We observe,

for $p_2 \in V(T_1(G)) - V(G)$, $d_{T_1(G)}(p_2) = d_G(w_i) + d_G(w_j)$ where $p_2 = w_iw_j \in E(G)$.

$$J_2 = n_H M_1(G) + 8e_H e_G + 2e_H M_1(G) + 2e_G M_1(H) + d_H(q_1)[d_G(w_i) + d_G(w_j)]$$

$$+ \sum_{q_1 \in V(H)} \sum_{w_iw_j \in E(G)} d_G(w_i) + d_G(w_j) + d_G(p_1)[d_G(w_i) + d_G(w_j)]$$

$$= 2n_H M_1(G) + 8e_H e_G + 4e_H M_1(G) + 2e_G M_1(H) + 2e_G n_H M_1(G).$$

$$J_3 = \sum_{q_1 \in V(H)} \sum_{p_1p_2 \in \alpha_1, p_1, p_2 \in \gamma_1} [d_{T_1(G)}(p_1) + d_{T_1(G)}(p_2)] + [d_{T_1(G)}(p_1)d_{T_1(G)}(p_2)]$$

$$= n_H \sum_{\substack{u_iu_j \in E(G), \\ u_ju_k \in E(G)}} [d_G(u_i) + d_G(u_j) + d_G(u_j) + d_G(u_k)]$$

$$+ [d_G(u_i) + d_G(u_j)][d_G(u_j) + d_G(u_k)]$$

$$= n_H \sum_{\substack{u_iu_j \in E(G), \\ u_ju_k \in E(G)}} d_G(u_i)[1 + d_G(u_k)] + d_G(u_k)[1 + d_G(u_j)]$$

$$+ d_G(u_j)[d_G(u_i) + d_G(u_j)].$$

Adding $J_1,\ J_2,\ J_3$ in (2) we get desired result. □

Theorem 3. *Let G and H be two connected graphs. Then*

$$GO_1(G +_{T_2} H) = 4n_H GO_1(G) + GO_1(H) + 8e_H M_1(G) + 5e_G M_1(H)$$

$$+ 6n_H M_1(G) + 4n_H M_2(G) + 24e_H e_G + 4n_H e_G.$$

Proof. We know that,

$$\begin{aligned}
GO_1(G +_{T_2} H) &= \sum_{(p_1,q_1)(p_2,q_2)\in E(G+_{T_2}H)} d_{G+_{T_2}H}(p_1,q_1) + d_{G+_{T_2}H}(p_2,q_2) \\
&+ d_{G+_{T_2}H}(p_1,q_1) d_{G+_{T_2}H}(p_1,q_2) \\
&= \sum_{p_1\in V(G)} \sum_{q_1q_2\in E(H)} d_{G+_{T_2}H}(p_1,q_1) + d_{G+_{T_2}H}(p_1,q_2) \\
&+ d_{G+_{T_2}H}(p_1,q_1) d_{G+_{T_2}H}(p_1,q_2) + d_{G+_{T_2}H}(p_2,q_1) \\
&+ \sum_{q_1\in V(H)} \sum_{p_1p_2\in E(T_1(G))} d_{G+_{T_2}H}(p_1,q_1) \\
&+ d_{G+_{T_2}H}(p_1,q_1) d_{G+_{T_2}H}(p_2,q_1) \\
&= K_1 + K_2. \qquad (3)
\end{aligned}$$

Where K_1 and K_1 are the sums of the above terms, in order

$$\begin{aligned}
K_1 &= \sum_{p_1\in V(G)} \sum_{q_1q_2\in E(H)} 2d_{T_2(G)}(p_1) + d_H(q_1) + d_H(q_2) + d^2_{T_2(G)}(p_1) \\
&+ d_{T_2(G)}(p_1)[d_H(q_1) + d_H(q_2)] + d_H(q_1)d_H(q_2) \\
&= \sum_{p_1\in V(G)} \sum_{q_1q_2\in E(H)} 4d_{(G)}(p_1) + d_H(q_1) + d_H(q_2) + 4d^2_G(p_1) \\
&+ 2d_{(G)}(p_1)[d_H(q_1) + d_H(q_2)] + d_H(q_1)d_H(q_2) \\
&= \sum_{p_1\in V(G)} 4e_H d_G(p_1) + GO_1(H) + 4e_H d^2_G(p_1) + 2d_G(p_1)M_1(H) \\
&= 8e_He_G + GO_1(H) + 4e_HM_1(G) + 4e_GM_1(H). \qquad (3a)
\end{aligned}$$

$\forall$ edge $p_1p_2 \in E(T_2(G))$ and vertex $q_1 \in V(H)$. Here we denote

$E(T_2(G)) = \alpha_2$, $V(G) = \beta$, $V(T_2(G)) - V(G) = \gamma_2$

$$\begin{aligned}
K_2 &= \sum_{q_1\in V(H)} \sum_{p_1p_2\in E(T_2(G))} d_{G+_{T_2}H}(p_1, q_1) + d_{G+_{T_2}H}(p_2, q_1) \\
&+ d_{G+_{T_2}H}(p_1, q_1)d_{G+_{T_2}H}(p_2, q_1) + \sum_{q_1\in V(H)} \sum_{\substack{p_1p_2\in\alpha_2,\\ p_1\in\beta,\\ p_2\in\gamma_2}} d_{G+_{T_2}H}(p_1, q_1) \\
&+ d_{G+_{T_2}H}(p_2, q_1) + d_{G+_{T_2}H}(p_1, q_1)d_{G+_{T_2}H}(p_2, q_1) \\
&= K_3 + K_4. \qquad (3b).
\end{aligned}$$

$\forall$ $q_1 \in V(H)$ and edge $p_1p_2 \in E(T_2(G))$ if and only if $p_1p_2 \in E(G)$.

$$\begin{aligned}
K_3 &= \sum_{q_1\in V(H)} \sum_{p_1p_2\in E(G)} d_{G+_{T_2(G)}H}(p_1,q_1) + d_{G+_{T_2(G)}H}(p_2,q_1) \\
&+ d_{G+_{T_2(G)}H}(p_1,q_1) d_{G+_{T_2(G)}H}(p_2,q_1) \\
&= \sum_{q_1\in V(H)} \sum_{p_1p_2\in E(G)} d_{T_2(G)}(p_1) + d_H(q_1) + d_{T_2(G)}(p_2) + d_H(q_1) \\
&+ [d_{T_2(G)}(p_1) + d_H(q_1)][d_{T_2(G)}(p_2) + d_H(q_1)] \\
&= \sum_{q_1\in V(H)} \sum_{p_1p_2\in E(G)} 2d_G(p_1) + 2d_H(q_1) + 2d_G(p_2) + 4d_G(p_1)d_G(p_2) \\
&+ 2d_G(p_1)d_H(q_1) + 2d_H(q_1)d_G(p_2) + d_H^2(q_1) \\
&= 4n_H GO_1(G) + 4e_H M_1(G) + e_G M_1(H) + 4n_H M_2(G) + 4e_H e_G.
\end{aligned}$$

Since we have $d_{T_2(G)}(a) = 2d_G(a)$ for each vertex $p_1 \in V(G)$ and $d_{T_2}(p_2) = 2$ for each vertex $p_2 \in V(T_2(G)) - V(G)$.

$$\begin{aligned}
K_4 &= \sum_{q_1\in V(H)} \sum_{\substack{p_1p_2\in\alpha_2,\\ p_1\in\beta,\\ p_2\in\gamma_2}} d_{T_2(G)}(p_1) + d_H(q_1) + d_{T_2(G)}(p_2) \\
&+ [d_{T_2(G)}(p_1) + d_H(q_1)] d_{T_2(G)}(p_2) \\
&= \sum_{q_1\in V(H)} \sum_{\substack{p_1p_2\in\alpha_2,\\ p_1\in\beta,\\ p_2\in\gamma_2}} d_{T_2(G)}(p_1) + d_H(q_1) + d_{T_2(G)}(p_2) \\
&+ d_{T_2(G)}(p_1) d_{T_2(G)}(p_2) + d_H(q_1) d_{T_2(G)}(p_2)
\end{aligned}$$

$$= \sum_{q_1 \in V(H)} \sum_{\substack{p_1p_2 \in \alpha_2, \\ p_1 \in \beta, \\ p_2 \in \gamma_2}} [6d_G(p_1) + 3d_H(q_1) + 2]$$
$$= \sum_{q_1 \in V(H)} \sum_{p_1 \in V(G)} d_G(p_1)[6d_G(p_1) + 3d_H(q_1) + 2]$$
$$= 6n_H M_1(G) + 12e_G e_H + 4n_H e_G.$$

Adding K_3 and K_4 and substitute in (3b) we get

$$4n_H GO_1(G) + 16e_H e_G + 6n_H M_1(G) + 4e_H M_1(G) + e_G M_1(H)$$
$$+4n_H M_2(G) + 4n_H e_G. \qquad (3c)$$

Substitute $(3a)$ and (3c) in (3) we get desired results.

$$GO_1(G +_{T_2} H) = 4n_H GO_1(G) + GO_1(H) + 8e_H M_1(G) + 5e_G M_1(H)$$
$$+ 6n_H M_1(G) + 4n_H M_2(G) + 24e_H e_G + 4n_H e_G.$$

□

Theorem 4. *Let G and H be two connected graphs. Then*

$$GO_1(G +_T H) = 4n_H GO_1(G) + n_G GO_1(H) + 12e_H M_1(G)$$

$$+ 6e_G M_1(H) + 2n_H M_1(G) + e_G M_2(H) + 8e_G M_1(G)$$
$$+ 20e_H e_G + n_H \sum_{\substack{q_iq_j \in E(G),\\ q_jq_k \in E(G)}} d_G(q_i) + 2d_G(q_j) + d_G(q_k)$$
$$+ [d_G(q_i) + d_G(q_j)][d_G(q_j) + d_G(q_k)].$$

Proof. Let,

$$\begin{aligned} GO_1(G +_T H) &= \sum_{(p_1,q_1)(p_2,q_2) \in E(G+_TH)} d_{G+_TH}(p_1,q_1) + d_{G+_TH}(p_2,q_2) \\ &+ d_{G+_TH}(p_1,q_1) d_{G+_TH}(p_2,q_2) \\ &= \sum_{p_1 \in V(G)} \sum_{q_1q_2 \in E(H)} d_{G+_TH}(p_1,q_1) + d_{G+_TH}(p_1,q_2) \\ &+ d_{G+_TH}(p_1,q_1) d_{G+_TH}(p_1,q_2) + \sum_{q_1 \in V(H)} \sum_{p_1p_2 \in E(T(G))} d_{G+_TH}(p_1,q_1) \\ &+ d_{G+_TH}(p_2,q_1) + d_{G+_TH}(p_1,q_1) d_{G+_TH}(p_2,q_1). \end{aligned}$$

Note that $E(T(G)) = E(G) \bigcup E(S(G)) \bigcup E(L(G))$

$$\begin{aligned} GO_1(G +_T H) &= \sum_{p_1 \in V(G)} \sum_{q_1q_2 \in E(H)} d_{G+_TH}(p_1,q_1) + d_{G+_TH}(p_1,q_2) \\ &+ d_{G+_TH}(p_1,q_1) d_{G+_TH}(p_1,q_2) + \sum_{q_1 \in V(H)} \sum_{\substack{(p_1p_2) \in E(T(G)),\\ (p_1,p_2) \in V(G)}} d_{G+_TH}(p_1,q_1) \\ &+ d_{G+_TH}(p_2,q_1) + d_{G+_TH}(p_1,q_1) d_{G+_TH}(p_2,q_1) \end{aligned}$$

$$
\begin{aligned}
&+ \sum_{q_1\in V(H)} \sum_{\substack{(p_1p_2)\in\alpha_3,\\ p_1\in\beta,\\ p_2\in\gamma_3}} d_{G+_TH}(p_1,q_1)+d_{G+_TH}(p_2,q_1)\\
&+ d_{G+_TH}(p_1,q_1)d_{G+_TH}(p_2,q_1)+\sum_{q_1\in V(H)} \sum_{\substack{(p_1p_2)\in\alpha_3,\\ (p_1,p_2)\in\gamma_3}} d_{G+_TH}(p_1,q_1)\\
&+ d_{G+_TH}(p_2,q_1)+d_{G+_TH}(p_1,q_1)d_{G+_TH}(p_2,q_1)\\
&= L_1+L_2+L_3+L_4. \qquad (4)
\end{aligned}
$$

where L_1, L_2, L_3, L_4 are the sums of the above terms, in order

$$
\begin{aligned}
L_1 &= \sum_{p_1\in V(G)} \sum_{q_1q_2\in E(H)} 2d_{T(G)}(p_1)+d_H(q_1)+d_H(q_2)\\
&+ [d_{T(G)}(p_1)+d_H(q_1)][d_{T(G)}(p_1)d_H(q_2)]\\
&= \sum_{p_1\in V(G)} \sum_{q_1q_2\in E(H)} 4d_G(p_1)+d_H(q_1)+d_H(q_2)+4d_G^2(p_1)+2d_G(p_1)d_H(q_1)\\
&+ 2d_G(p_1)d_H(q_2)+d_H(q_1)d_H(q_2)\\
&= n_GGO_1(H)+4e_HM_1(G)+4e_GM_1(H)+8e_Ge_H.
\end{aligned}
$$

$$
\begin{aligned}
L_2 &= \sum_{q_1\in V(H)} \sum_{p_1p_2\in\alpha_3, p_1,p_2\in\beta} d_{T(G)}(p_1)+2d_H(q_1)+d_{T(G)}(p_2)\\
&+ [d_{T(G)}(p_1)+d_H(q_1)][d_{T(G)}(p_2)d_H(q_1)]\\
&= \sum_{q_1\in V(H)} \sum_{p_1p_2\in E(G)} 2d_G(p_1)+2d_G(p_2)+2d_H(q_1)+d_H^2(q_1)+2d_G(p_2)d_H(q_1)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 4d_G(p_1)d_G(p_2) + 2d_G(p_1)d_H(q_1) \\
&= \quad 2n_H GO_1(G) + 4e_H M_1(G) + e_G M_2(H) + 2n_H M_2(G) + 4e_G e_H.
\end{aligned}
$$

$$
\begin{aligned}
L_3 &= \sum_{q_1 \in V(H)} \sum_{\substack{p_1p_2 \in \alpha_3, \\ p_1 \in \beta, \\ p_2 \in \gamma_3}} [d_{T(G)}(p_1) + d_{T(G)}(p_2) + 2d_H(q_1)] \\
&+ [d_{T(G)}(p_1) + d_H(q_1)][d_{T(G)}(p_2)d_H(q_1)] \\
&= \sum_{q_1 \in V(H)} \sum_{(p_1 \in V(G)} d_G(p_1)(2d_G(p_1) + d_H(q_1) + d_H(q_1) + d_G(p_1)d_H(q_1) + d_H^2(q_1) \\
&+ \sum_{q_1 \in V(H)} \sum_{\substack{p_1p_2 \in \alpha_3, \\ p_1 \in \beta, \\ p_2 \in \gamma_3}} d_{T(G)}(p_2) + 2d_G(p_1)d_{T(G)}(p_2) + d_H(q_1)d_{T(G)}(p_2).
\end{aligned}
$$

Note that $p_2 \in V(T(G)) - V(G)$, $d_{T(G)}(p_2) = d_G(p) + d_G(q)$

where $p_2 = pq \in E(G)$

$$
\begin{aligned}
&= \quad 2n_H M_1(G) + 4e_H M_1(G) + 2e_G M_1(H) + 8e_H e_G + \sum_{q_1 \in V(H)} \sum_{\substack{p_1 \in \beta, \\ p_2 \in \gamma_3}} (d_G(p) + d_G(q)) \\
&+ \quad 2d_G(p_1)(d_G(p) + d_G(q)) + d_H(q_1)(d_G(p) + d_G(q)) \\
&= \quad 2n_H M_1(G) + 4e_H M_1(G) + 2e_G M_1(H) + 2n_H M_1(G) + 8e_G M_1(G) + 4e_H M_1(G) \\
&= \quad 4n_H M_1(G) + 4e_H M_1(G) + 8e_G M_1(G) + 2e_G M_1(H) + 8e_H e_G.
\end{aligned}
$$

$$
\begin{aligned}
L_4 &= \sum_{q_1\in V(H)} \sum_{(p_1,p_2)\in\gamma_3} d_{G+_TH}(p_1,q_1) + d_{G+_TH}(p_2,q_1) + d_{G+_TH}(p_1,q_1)d_{G+_TH}(p_2,q_1) \\
&= \sum_{q_1\in V(H)} \sum_{p_1,p_2\in\gamma_3} d_{T(G)}(p_1) + d_{T(G)}(p_2) + d_{T(G)}(p_1)d_{T(G)}(p_2) \\
&= n_H \sum_{q_iq_j\in E(G),q_jq_k\in E(G)} (d_G(q_i) + d_G(q_j)) + (d_G(q_j) + d_G(q_k)) \\
&+ [d_G(q_i) + d_G(q_j)][d_G(q_j) + d_G(q_k)].
\end{aligned}
$$

Adding L_1, L_2, L_3, L_4 in (4) we get required result. □

Chapter 3

Some adratic indices of Dutch windmill graph using graph operator

3.1 Introduction

The Dutch windmill graph is denoted by D_n^m and it is the graph obtained by taking m copies of the cycle C_n with a vertex in common [30]. It contains $(n-1)m+1$ vertices and mn edges.

V.Lokesha and et. al., $[28, 45]$ are discussed on the operators and nano structures. Motivated from this, we computed Dutch windmill graph of certain graph operators using adriatic indices.

3.2 On discrete adriatic indices of a subdivision-Dutch windmill graph

Theorem 5. *Let $S(D_n^m)$ be a subdivision formed by dutch windmill graph then*

1. $Adr(S(D_n^m))_{\zeta_1(x,y)} = 2m\Big((n-1)(log2)^2 + log(2)log(2m)\Big)$, *if* $\mu_{i,a} = \mu_{1,1}$

2. $$Adr(S(D_n^m))_{\zeta_2(x,y)} = \begin{cases} \dfrac{m(n-1)}{\sqrt{log(2)}} + \dfrac{2m}{\sqrt{log(2)} + \sqrt{log(2m)}}, & \text{if } \mu_{i,a} = \mu_{1,1/2} \\ 2m(n-1) + \dfrac{4m^2}{1+m}, & \text{if } \mu_{2,-1} \end{cases}$$

3. $$Adr(S(D_n^m))_{\zeta_3(x,y)} = \begin{cases} 2m \mid log2 - log2m \mid, & \text{if } \mu_{i,a} = \mu_{1,1} \\ 2m \mid log^2(2) - log^2(2m) \mid, & \text{if } \mu_{1,2} \\ 2m \mid \sqrt{2}(1-\sqrt{m}) \mid, & \text{if } \mu_{2,1/2} \\ 2m \mid 2(1-m) \mid, & \text{if } \mu_{2,1} \\ \frac{m}{2} \mid \frac{m-1}{n2} \mid, & \text{if } \mu_{3,1/2} \end{cases}$$

4. $Adr(S(D_n^m))_{\zeta_4(x,y)} = \begin{cases} \mid m-1 \mid, & \text{if } \mu_{i,a} = \mu_{2,-1} \\ 2\sqrt{2}\frac{\sqrt{m}-1}{\sqrt{m}}, & \text{if } \mu_{2,-1/2} \end{cases}$

5. $Adr(S(D_n^m))_{\zeta_5(x,y)} = 2(mn - m + \sqrt{m}), \text{if } \mu_{i,a} = \mu_{2,1/2}$

6. $Adr(S(D_n^m))_{\zeta_6(x,y)} = \begin{cases} 2m(\sqrt{m} + n - 1), & \text{if } \mu_{i,a} = \mu_{2,1/2} \\ 2m(m + n - 1), & \text{if } \mu_{2,1} \\ 2m(m^2 + n - 1), & \text{if } \mu_{2,2} \end{cases}$

7. $Adr(S(D_n^m))_{\zeta_7(x,y)} = 2m^2 + 4mn - 4m + 2, \text{if } \mu_{i,a} = \mu_{2,1}$

Proof. We define the edge set of $S(D_n^m)$ with their vertices degrees. There are two types of edges with respect to degrees of end vertices in $S(D_n^m)$, namely the degrees of end vertices $(2, 2)$ and degrees of end vertices $(2, 2m)$. Thus, we have shown in the following Table 3.1.

Table 3.1: The edge partition of the edges of $S(D_n^m)$ based on degrees of end vertices

$E_{\{d(u),d(v)\}}$	$E_{(2,2)}$	$E_{(2,2m)}$
Number of Edges	$2m(n-1)$	$2m$

We presented these partitions with their edge cardinalities in Table 3.1. Hence utilizing the adriatic indices definitions, we obtained required results. □

3.3 On discrete adriatic indices of a derived-Dutch windmill graph

Theorem 6. *Let* $(D_n^m)^\dagger$ *be derived graph formed by Dutch windmill graph then*

1. $Adr(D_n^m)^\dagger_{\zeta_1(x,y)} = \frac{[(n-1)m+1](n-1)m}{2}(log(n-1)m)^2, \textit{if } \mu_{i,a} = \mu_{1,1}$

2. $$Adr(D_n^m)^\dagger_{\zeta_2(x,y)} = \begin{cases} \dfrac{[(n-1)m+1](n-1)m}{4\sqrt{log(n-1)m}}, & \textit{if } \mu_{i,a} = \mu_{1,1/2} \\ \\ \dfrac{[(n-1)m+1](n-1)^2m^2}{4}, & \textit{if } \mu_{2,-1} \end{cases}$$

3. $Adr(D_n^m)^\dagger_{\zeta_3(x,y)} = 0, \textit{if } \mu_{i,a} = \mu_{1,1},\ \mu_{1,2},\ \mu_{2,1/2},\ \mu_{2,1} \textit{ and } \mu_{3,1/2}$

4. $Adr(D_n^m)^\dagger_{\zeta_4(x,y)} = 0, \textit{if } \mu_{i,a} = \mu_{2,-1} \textit{ and } \mu_{2,-1/2}$

5. $Adr(D_n^m)^{\dagger}_{\zeta_5(x,y)} = \frac{[(n-1)m+1](n-1)m}{2}$, *if* $\mu_{i,a} = \mu_{2,1/2}$

6. $Adr(D_n^m)^{\dagger}_{\zeta_6(x,y)} = \frac{[(n-1)m+1](n-1)m}{2}$, *if* $\mu_{i,a} = \mu_{2,1/2}$, $\mu_{2,1}$ *and* $\mu_{2,2}$

7. $Adr(D_n^m)^{\dagger}_{\zeta_7(x,y)} = [(n-1)m+1](n-1)m$, *if* $\mu_{i,a} = \mu_{2,1}$

Proof. Consider the Dutch windmill graph D_n^m. Splitting the edges of the type $E_{(d_u,d_v)}$ where uv is an edge. In derived graph of D_n^m we get edge of the type $E_{((n-1)m,(n-1)m)}$. The number of edges of these types are given in the Table 3.2.

Table 3.2: The edge partition of the edges of $(D_n^m)^{\dagger}$ based on degrees of end vertices

$E_{\{d(u),d(v)\}}$	$E_{((n-1)m,(n-1)m)}$
Number of Edges	$((n-1)m+1)(n-1)m/2$

Using the above cardinalities of E and the definitions of adriatic indices, we get desired results. □

3.4 On discrete adriatic indices of a line-Dutch windmill graph

Theorem 7. *Let $L(D_n^m)$ be line graph formed by Dutch windmill graph then*

1. $Adr(L(D_n^m))_{\zeta_1(x,y)} = mlog(2)\Big((n-3)log(2)+2log(2)\Big)+m(2m-1)(log(2m))^2$, *if* $\mu_{i,a} = \mu_{1,1}$.

2. $$Adr(L(D_n^m))_{\zeta_2(x,y)} = \begin{cases} m\left(\frac{n-3}{2\sqrt{log2}} + \frac{2}{\sqrt{log(2)}+\sqrt{log(2m)}} + \frac{2m-1}{2\sqrt{log(2m)}}\right), \\ m^2(2m+n\frac{4}{1+m}-4), \end{cases}$$

3. $$Adr(L(D_n^m))_{\zeta_3(x,y)} = \begin{cases} 2m \mid log(m) \mid, & if\ \mu_{i,a} = \mu_{1,1} \\ 2m \mid log^2(2) - log^2(2m) \mid, & if\ \mu_{1,2} \\ 2\sqrt{2}m \mid (1-\sqrt{m}) \mid, & if\ \mu_{2,1/2} \\ 4m \mid (1-m) \mid, & if\ \mu_{2,1} \\ 2m \mid \frac{1}{2^2} - \frac{1}{2}^{2m} \mid, & if\ \mu_{3,1/2} \end{cases}$$

4. $Adr(L(D_n^m))_{\zeta_4(x,y)} = \begin{cases} \mid m-1 \mid, & \textit{if } \mu_{i,a} = \mu_{2,-1} \\ 2\sqrt{2}m \mid \frac{\sqrt{m}-1}{\sqrt{m}} \mid, & \textit{if } \mu_{2,-1/2} \end{cases}$

5. $Adr(L(D_n^m))_{\zeta_5(x,y)} = m(2m-n-4) + 2\sqrt{m}, \textit{if } \mu_{i,a} = \mu_{2,1/2}$

6. $Adr(L(D_n^m))_{\zeta_6(x,y)} = \begin{cases} m(2m + 2m^{\frac{1}{2}} + n - 4), & \textit{if } \mu_{i,a} = \mu_{2,1/2} \\ m(4m + n - 4), & \textit{if } \mu_{2,1} \\ m(2m + 2m^2 + n - 4), & \textit{if } \mu_{2,2} \end{cases}$

7. $Adr(L(D_n^m))_{\zeta_7(x,y)} = 2m(m^2 + 2m + n - 3), \textit{if } \mu_{i,a} = \mu_{2,1}$

Proof. We define the partitions of the edge set of $L(D_n^m)$ with respect to degree of vertices. There are three types of edges with respect to degrees of end vertices in $L(D_n^m)$ namely, $(2,2)$, $(2,2m)$, and $(2m,2m)$. Thus, we have shown in the following Table 3.3.

Table 3.3: The edge partition of the edges of $L(D_n^m)$ based on degrees of end vertices

$E_{\{d(u),d(v)\}}$	$E_{(2,2)}$	$E_{(2,2m)}$	$E_{(2m,2m)}$
Number of Edges	$m(n-3)$	$2m$	$m(2m-1)$

We presented these partitions with their cardinalities of E in Table 3.3. Hence utilizing the adriatic indices definitions, we obtained required results. □

Chapter 4

Adriatic indices and Sanskruti index envisage of carbon nanocone

4.1 Introduction and Preliminaries

The central part of graphical structure of carbon nanocone $CNC_m[n]$ [40] have a cycle of m-length and at the conical exterior around its central part n-levels of hexagons are positioned. The edge and vertex sets of carbon nanocone are $E(CNC_m[n]) = \frac{m(n+1)(3n+2)}{2}$ and $V(CNC_m[n]) = m(n+1)^2$ respectively, where $n \geq 1$, $m = 3, 4, 5, ...$ One can see [16, 22, 26, 36, 51] for relevant work on carbon nanomaterials.

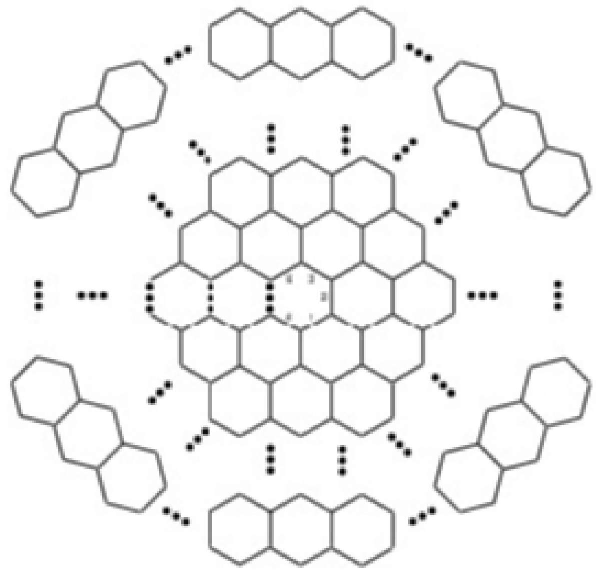

Figure 4.1: Carbonnanocone

4.2 Topological indices of carbon nanocone graph

Table 4.1: The edge partition of carbon nanocone $CNC_m[n]$ based on degrees of end vertices of each edge.

Number of edges	$(d_u, d_v), uv \in E(G)$
m	$(2, 2)$
$mn(3m + 1)/2$	$(3, 3)$
$2mn$	$(3, 2)$

Theorem 8. *Let G be a graph of $CNC_m[n]$ nanocones for $n = 1, 2, 3, ...$ and $m \geq 3$. Then*

$$
\begin{aligned}
ISI[G] &= \tfrac{m}{20}[45n^2 + 27n + 20]. \\
AZI[G] &= 8m + \tfrac{3^6}{2^7}mn(3n+1) + 16mn. \\
SDD[G] &= m\left[2 + n\left(3n + \tfrac{16}{3}\right)\right].
\end{aligned}
$$

Proof. The graph G consists of $m(n+1)^2$ vertices and $\frac{m(n+1)(3n+2)}{2}$ edges as shown in Figure 4.1. Using Table 4.1 we obtain the results as follows.

$$
\begin{aligned}
\therefore ISI[G] &= m\left[\tfrac{2.2}{2+2}\right] + \tfrac{mn(3n+1)}{2}\left[\tfrac{3.3}{3+3}\right] + 2mn\left[\tfrac{3.2}{3+2}\right] \\
&= \tfrac{m}{20}[45n^2 + 27n + 20]. \\
AZI[G] &= m\left[\tfrac{2.2}{2+2-2}\right]^3 + \tfrac{mn(3n+1)}{2}\left[\tfrac{3.3}{3+3-2}\right]^3 + 2mn\left[\tfrac{3.2}{3+2-2}\right]^3 \\
&= 8m + \tfrac{3^6}{2^7}mn(3n+1) + 16mn. \\
SDD[G] &= m\left[\tfrac{2^2+2^2}{2.2}\right] + \tfrac{mn(3n+1)}{2}\left[\tfrac{3^2+3^2}{3.3}\right] + 2mn\left[\tfrac{3^2+2^2}{3.2}\right] \\
&= m\left[2 + n\left(3n + \tfrac{16}{3}\right)\right].
\end{aligned}
$$

□

Theorem 9. *Let G be a graph of $CNC_m[n]$ nanocones for $n = 1, 2, 3, ...$ and $m \geq 3$. Then*

$$\begin{aligned} LM_1[G] &= 2m\left[ln2 + \tfrac{n(3n+1)}{3}ln3 + \tfrac{n}{3}[ln72]\right]. \\ \overline{LM_1}[G] &= ln[4.(6)^{\frac{n(3n+1)}{2}}5^{2n}]^m. \\ MLD[G] &= 2mn|ln(\tfrac{3}{2})|. \\ MLSD[G] &= 2mn|ln^2 3 - ln^2 2|. \end{aligned}$$

Proof. The graph G consists of $m(n+1)^2$ vertices and $\frac{m(n+1)(3n+2)}{2}$ edges as shown in Figure 4.1. Using Table 4.1 we obtain the results as follows.

$$\begin{aligned} \therefore LM_1[G] &= 2m\left[\tfrac{ln2}{2} + \tfrac{ln2}{2}\right] + 2\left[\tfrac{mn(3n+1)}{2}\right]\left[\tfrac{ln3}{3} + \tfrac{ln3}{3}\right] + 2(2mn)\left[\tfrac{ln2}{2} + \tfrac{ln3}{3}\right] \\ &= 2m\left[ln2 + \tfrac{n(3n+1)}{3}ln3 + \tfrac{n}{3}[ln72]\right]. \\ \overline{LM_1}[G] &= m[ln(2+2)] + \tfrac{mn(3n+1)}{2}[ln(3+3)] + 2mn[ln(3+2)] \\ &= ln[4.(6)^{\frac{n(3n+1)}{2}}5^{2n}]^m. \\ MLD[G] &= m|ln2 - ln2| + \tfrac{mn(3n+1)}{2}|ln3 - ln3| + 2mn|ln3 - ln2| \\ &= 2mn|ln\left(\tfrac{3}{2}\right)|. \\ MLSD[G] &= m|ln^2 2 - ln^2 2| + \tfrac{mn(3n+1)}{2}|ln^2 3 - ln^2 3| + 2mn|ln^2 3 - ln^2 2| \\ &= 2mn|ln^2 3 - ln^2 2|. \end{aligned}$$

□

Theorem 10. *Let G be a graph of $CNC_m[n]$ for $n = 1, 2, 3, ...$ and $m \geq 3$. Then*

$$\begin{aligned} MRD[G] &= 2mn|\sqrt{3} - \sqrt{2}|. \\ MIRD[G] &= 2mn|\tfrac{1}{\sqrt{3}} - \tfrac{1}{\sqrt{2}}|. \\ ISLSD[G] &= m\left[\tfrac{1}{2\sqrt{2}}\right] + \tfrac{mn(3n+1)}{2}\left[\tfrac{1}{2\sqrt{3}}\right] + 2mn\left[\tfrac{1}{\sqrt{3}+\sqrt{2}}\right]. \end{aligned}$$

Proof. The graph G consists of $m(n+1)^2$ vertices and $\frac{m(n+1)(3n+2)}{2}$ edges as shown in Figure 4.1. Using Table 4.1 we obtain the results as follows.

$$\begin{aligned} \therefore MRD[G] &= m|\sqrt{2} - \sqrt{2}| + \tfrac{mn(3n+1)}{2}|\sqrt{3} - \sqrt{3}| + 2mn|\sqrt{3} - \sqrt{2}| \\ &= 2mn|\sqrt{3} - \sqrt{2}|. \\ MIRD[G] &= m|\tfrac{1}{\sqrt{2}} - \tfrac{1}{\sqrt{2}}| + \tfrac{mn(3n+1)}{2}|\tfrac{1}{\sqrt{3}} - \tfrac{1}{\sqrt{3}}| + 2mn|\tfrac{1}{\sqrt{3}} - \tfrac{1}{\sqrt{2}}| \\ &= 2mn|\tfrac{1}{\sqrt{3}} - \tfrac{1}{\sqrt{2}}|. \\ ISLSD[G] &= m\left[\tfrac{1}{\sqrt{2}+\sqrt{2}}\right] + \tfrac{mn(3n+1)}{2}\left[\tfrac{1}{\sqrt{3}+\sqrt{3}}\right] + 2mn\left[\tfrac{1}{\sqrt{3}+\sqrt{2}}\right] \\ &= m\left[\tfrac{1}{2\sqrt{2}}\right] + \tfrac{mn(3n+1)}{2}\left[\tfrac{1}{2\sqrt{3}}\right] + 2mn\left[\tfrac{1}{\sqrt{3}+\sqrt{2}}\right]. \end{aligned}$$

□

4.3 On Sanskruti index of carbon nanocone

Table 4.2: The edge partition of carbon nanocone $CNC_m[n]$ based on degree sum of neighbourhood vertices of end vertices of each edge.

Number of edges	(S_u, S_v), $uv \in E(G)$
m	$(5,5)$
$2m$	$(5,7)$
$m(2n-2)$	$(6,7)$
mn	$(7,9)$
$(mn/2)(3n-1)$	$(9,9)$

Theorem 11. *Let G be a graph of $CNC_m[n]$ for $n = 1, 2, 3, ...$ and $m \geq 3$. Then*

$$\mathcal{S}[G] = \left[\frac{81}{16}\right]^3 + mn\left[\frac{9}{2}\right]^3 + m\left[\frac{25}{8}\right]^3 + 2m\left[\frac{7}{2}\right]^3 + m(2n-2)\left[\frac{42}{11}\right]^3.$$

Proof. The graph G consists of $m(n+1)^2$ vertices and $\frac{m(n+1)(3n+2)}{2}$ edges as shown in Figure 4.1. Using Table 4.2 we obtain the results as follows.

$$\begin{aligned}
\therefore \mathcal{S}[G] &= \frac{mn}{2}(3n-1)\left[\frac{9.9}{9+9-2}\right]^3 + mn\left[\frac{7.9}{7+9-2}\right]^3 + m\left[\frac{5.5}{5+5-2}\right]^3 + 2m\left[\frac{5.7}{5+7-2}\right]^3 \\
&+ m(2n-2)\left[\frac{6.7}{6+7-2}\right]^3 \\
&= \left[\frac{81}{16}\right]^3 + mn\left[\frac{9}{2}\right]^3 + m\left[\frac{25}{8}\right]^3 + 2m\left[\frac{7}{2}\right]^3 + m(2n-2)\left[\frac{42}{11}\right]^3.
\end{aligned}$$

□

4.4 On Inverse sum indeg and symmetric division deg indices of a semi-total point graph of carbon nanocone

Table 4.3: The edge partition of semi-total point graph of carbon nanocone $R[CNC_m[n]]$.

Number of edges	(d_u, d_v),$uv \in E(G)$
$2m(n+1)$	$(2,4)$
m	$(4,4)$
$3mn(n+1)$	$(2,6)$
$2mn$	$(4,6)$
$(mn/2)(3n+1)$	$(6,6)$

Theorem 12. *Let G be a graph of $R[CNC_m[n]]$ nanocones for $n = 1, 2, 3, ...$ and $m \geq 3$. Then*

$$\begin{aligned} ISI[G] &= m\left[9n^2 + \tfrac{359}{30} + \tfrac{37}{6}\right]. \\ SDD[G] &= m\left[24n^2 + \tfrac{526}{15}n + \tfrac{32}{3}\right]. \end{aligned}$$

Proof. The graph G consists of $\frac{m(n+1)(5n+4)}{2}$ vertices and $\frac{3m}{2}(n+1)(3n+2)$ edges by definition $R(G)$. Using Table 4.3 we obtain the results as follows.

$$\begin{aligned}
\therefore ISI[G] &= 2m(n+1)\left[\tfrac{2.4}{2+4}\right] + m\left[\tfrac{4.4}{4+4}\right] + 3mn(n+1)\left[\tfrac{2.6}{2+6}\right] + 2mn\left[\tfrac{4.6}{4+6}\right] \\
&+ \tfrac{mn}{2}(3n+1)\left[\tfrac{6.6}{6+6}\right] \\
&= m\left[9n^2 + \tfrac{359}{30} + \tfrac{37}{6}\right]. \\
SDD[G] &= 2m(n+1)\left[\tfrac{2^2+4^2}{2+4}\right] + m\left[\tfrac{4^2+4^2}{4+4}\right] + 3mn(n+1)\left[\tfrac{2^2+6^2}{2+6}\right] \\
&+ 2mn\left[\tfrac{4^2+6^2}{4+6}\right] + \tfrac{mn}{2}(3n+1)\left[\tfrac{6^2+6^2}{6+6}\right] \\
&= m\left[24n^2 + \tfrac{526}{15}n + \tfrac{32}{3}\right].
\end{aligned}$$

□

Chapter 5

Operations Of triglyceride via adriatic indices

5.1 Introduction

Triglycerides are a kind of lipid found in human blood [37]. Level of triglyceride increase the peril of cardinal disease, as reported by American Heart Association.

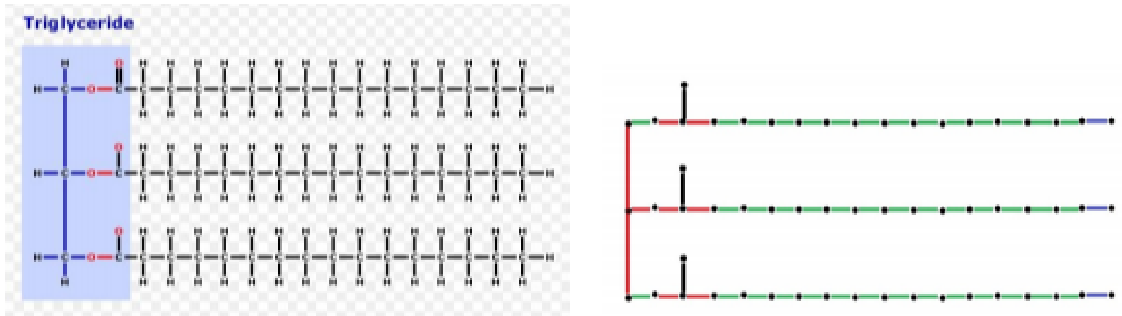

Figure 5.1: Moleculer and 2D structure of Triglyceride

5.2 Total graph of triglyceride

Table 5.1: The edge partition of total graph of triglyceride.

(d_u, d_v),$uv \in E(G)$	Number of edges
$(2,3)$	3
$(2,4)$	6
$(2,6)$	3
$(3,4)$	6
$(4,4)$	159
$(4,5)$	22
$(4,6)$	12
$(5,5)$	7
$(5,6)$	9

Theorem 13. *Let G be a total graph of triglyceride. Then*

$$ISI[G] = 464.12.$$

$$AZI[G] = 31415.57547.$$

$$SDD[G] = 465.4.$$

Proof. The graph G be a total graph of triglyceride which consists of 9 different type of edge sets from Table 5.1. Using from the Table 5.1 we computed respective index as follows.

$$\begin{aligned}
\therefore ISI[G] &= 3\left[\frac{2.3}{2+3}\right]+6\left[\frac{2.4}{2+4}\right]+3\left[\frac{2.6}{2+6}\right] \\
&+ 6\left[\frac{3.4}{3+4}\right]+159\left[\frac{4.4}{4+4}\right]+22\left[\frac{4.5}{4+5}\right] \\
&+ 12\left[\frac{4.6}{4+6}\right]+7\left[\frac{5.5}{5+5}\right]+9\left[\frac{5.6}{5+6}\right] \\
&= 464.12. \\
AZI[G] &= 3\left[\frac{2.3}{2+3-2}\right]^3+6\left[\frac{2.4}{2+4-2}\right]^3+3\left[\frac{2.6}{2+6-2}\right]^3 \\
&+ 6\left[\frac{3.4}{3+4-2}\right]^3+159\left[\frac{4.4}{4+4-2}\right]^3+22\left[\frac{4.5}{4+5-2}\right]^3 \\
&+ 12\left[\frac{4.6}{4+6-2}\right]^3+7\left[\frac{5.5}{5+5-2}\right]^3+9\left[\frac{5.6}{5+6-2}\right]^3 \\
&= 31415.57547. \\
SDD[G] &= 3\left[\frac{2^2+3^2}{2.3}\right]+6\left[\frac{2^2+4^2}{2.4}\right]+3\left[\frac{2^2+6^2}{2.6}\right] \\
&+ 6\left[\frac{3^2+4^2}{3.4}\right]+159\left[\frac{4^2+4^2}{4.4}\right]+22\left[\frac{4^2+5^2}{4.5}\right] \\
&+ 12\left[\frac{4^2+6^2}{4.6}\right]+7\left[\frac{5^2+5^2}{5.5}\right]+9\left[\frac{5^2+6^2}{5.6}\right]. \\
&= 465.4.
\end{aligned}$$

□

Theorem 14. *Let G be a total graph of triglyceride. Then*

$$
\begin{aligned}
H[G] &= 55.7395. \\
SCI[G] &= 79.3898. \\
MD[G] &= 88.
\end{aligned}
$$

Proof. The graph G be a total graph of triglyceride which consists of 9 different type of edge sets from Table 5.1. Using from the Table 5.1 we computed respective index as follows.

$$
\begin{aligned}
\therefore H[G] &= 3\left[\frac{2}{2+3}\right]+6\left[\frac{2}{2+4}\right]+3\left[\frac{2}{2+6}\right] \\
&+ 6\left[\frac{2}{3+4}\right]+159\left[\frac{2}{4+4}\right]+22\left[\frac{2}{4+5}\right] \\
&+ 12\left[\frac{2}{4+6}\right]+7\left[\frac{2}{5+5}\right]+9\left[\frac{2}{5+6}\right] \\
&= 55.7395. \\
SCI[G] &= 3\left[\frac{1}{\sqrt{2+3}}\right]+6\left[\frac{1}{\sqrt{2+4}}\right]+3\left[\frac{1}{\sqrt{2+6}}\right] \\
&+ 6\left[\frac{1}{\sqrt{3+4}}\right]+159\left[\frac{1}{\sqrt{4+4}}\right]+22\left[\frac{1}{\sqrt{4+5}}\right] \\
&+ 12\left[\frac{1}{\sqrt{4+6}}\right]+7\left[\frac{1}{\sqrt{5+5}}\right]+9\left[\frac{1}{\sqrt{5+6}}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= 79.3898. \\
MD[G] &= 3|2-3| + 6|2-4| + 3|2-6| \\
&+ 6|3-4| + 159|4-4| + 22|4-5| \\
&+ 12|4-6| + 7|5-5| + 9|5-6| \\
&= 88.
\end{aligned}
$$

□

Theorem 15. *Let G be a total graph of triglyceride. Then*

$$
\begin{aligned}
MRD[G] &= 21.6899. \\
MIRD[G] &= 5.6056. \\
MHD[G] &= 3.9688.
\end{aligned}
$$

Proof. The graph G be a total graph of triglyceride which consists of 9 different type of edge sets from Table 5.1. Using from the Table 5.1 we computed respective index as follows.

$$
\begin{aligned}
\therefore MRD[G] &= 3|\sqrt{2}-\sqrt{3}| + 6|\sqrt{2}-\sqrt{4}| + 3|\sqrt{2}-\sqrt{6}| \\
&+ 6|\sqrt{3}-\sqrt{4}| + 159|\sqrt{4}-\sqrt{4}| + 22|\sqrt{4}-\sqrt{5}|
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 12|\sqrt{4}-\sqrt{6}|+7|\sqrt{5}-\sqrt{5}|+9|\sqrt{5}-\sqrt{6}| \\
&= \quad 21.6899. \\
MIRD[G] &= 3\left|\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right|+6\left|\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{4}}\right|+3\left|\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{6}}\right| \\
&+ 6\left|\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}\right|+159\left|\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{4}}\right|+22\left|\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{5}}\right| \\
&+ 12\left|\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{6}}\right|+7\left|\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{5}}\right|+9\left|\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{6}}\right| \\
&= 5.6056. \\
MHD[G] &= 3|2^{-2}-2^{-3}|+6|2^{-2}-2^{-4}|+3|2^{-2}-2^{-6}| \\
&+ 6|2^{-3}-2^{-4}|+159|2^{-4}-2^{-4}|+22|2^{-4}-2^{-5}| \\
&+ 12|2^{-4}-2^{-6}|+7|2^{-5}-2^{-5}|+9|2^{-5}-2^{-6}| \\
&= 3.9688.
\end{aligned}
$$

□

5.3 Subdivision graph of triglyceride

Table 5.2: The edge partition of Subdivision graph of Triglyceride.

(d_u, d_v),$uv \in E(G)$	Number of edges
$(1, 2)$	6
$(2, 2)$	94
$(2, 3)$	12

Theorem 16. *Let G be a subdivision graph of triglyceride. Then*

$$\begin{aligned} ISI[G] &= 112.4. \\ AZI[G] &= 840.5. \\ SDD[G] &= 229. \end{aligned}$$

Proof. The graph G be a subdivision graph of triglyceride which consists of 3 different type of edge sets from Table 5.2. Using from the Table 5.2 we computed respective index as follows.

$$\begin{aligned} \therefore ISI[G] &= 6\left[\frac{1.2}{1+2}\right] + 94\left[\frac{2.2}{2+2}\right] + 12\left[\frac{2.3}{2+3}\right] \\ &= 112.4. \\ AZI[G] &= 6\left[\frac{1.2}{1+2-2}\right]^3 + 94\left[\frac{2.2}{2+2-2}\right]^3 + 12\left[\frac{1.3}{1+3-2}\right]^3 \\ &= 840.5 \\ SDD[G] &= 6\left[\frac{1^2+2^2}{1.2}\right] + 94\left[\frac{2^2+2^2}{2.2}\right] + 12\left[\frac{2^2+3^2}{2.3}\right] \\ &= 229. \end{aligned}$$

□

Theorem 17. *Let G be a subdivision graph of triglyceride. Then*

$$H[G] = 55.8.$$

$$SCI[G] = 55.83066476.$$

$$MD[G] = 18.$$

Proof. The graph G be a subdivision graph of triglyceride which consists of 3 different type of edge sets from Table 5.2. Using from the Table 5.2 we computed respective index as follows.

$$\begin{aligned} \therefore H[G] &= 6\left[\frac{2}{1+2}\right] + 94\left[\frac{2}{2+2}\right] + 12\left[\frac{2}{2+3}\right] \\ &= 55.8. \\ SCI[G] &= 6\left[\frac{1}{\sqrt{1+2}}\right] + 94\left[\frac{1}{\sqrt{2+2}}\right] + 12\left[\frac{1}{\sqrt{2+3}}\right] \\ &= 55.83066476. \\ MD[G] &= 6|1-2| + 94|2-2| + 12|2-3| \\ &= 18. \end{aligned}$$

□

Theorem 18. *Let G be a subdivision graph of triglyceride. Then*

$$MRD[G] = 6.299328317.$$

$$MIRD[G] = 3.314437457.$$

$$MHD[G] = 3.$$

Proof. The graph G be a subdivision graph of triglyceride which consists of 3 different type of edge sets from Table 5.2. Using from the Table 5.2 we computed respective index as follows.

$$\begin{aligned}
\therefore MRD[G] &= 6|\sqrt{2}-\sqrt{1}| + 94|\sqrt{2}-\sqrt{2}| + 12|\sqrt{3}-\sqrt{2}| \\
&= 6.299328317. \\
MIRD[G] &= 6\left|\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{2}}\right| + 94\left|\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right| + 12\left|\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right| \\
&= 3.314437457. \\
MHD[G] &= 6|2^{-1}-2^{-2}| + 94|2^{-2}-2^{-2}| + 12|2^{-2}-2^{-3}| \\
&= 3.
\end{aligned}$$

□

5.4 Semi-total point graph of triglyceride

Table 5.3: The edge partition of additional subdivision graph $R(G)$ of triglyceride.

(d_u, d_v),$uv \in E(G)$	Number of edges
$(2,6)$	15
$(2,2)$	6
$(2,4)$	97

(d_u, d_v),$uv \in E(G)$	Number of edges
$(4,4)$	41
$(4,6)$	9

Theorem 19. *Let G be a Semi-total point graph $R(G)$ of triglyceride. Then*

$$\begin{aligned} ISI[G] &= 261.4333333. \\ AZI[G] &= 1964.481481. \\ SDD[G] &= 406. \end{aligned}$$

Proof. The graph G be semi-total point graph of triglyceride which consists of 5 different type of edge sets from Table 5.3. Using from the Table 5.3 we computed respective index as follows.

$$\begin{aligned} \therefore ISI[G] &= 6\left[\frac{2.2}{2+2}\right] + 15\left[\frac{2.6}{2+6}\right] + 97\left[\frac{2.4}{2+4}\right] + 41\left[\frac{4.4}{4+4}\right] + 9\left[\frac{4.6}{4+6}\right] \\ &= 261.4333333. \\ AZI[G] &= 6\left[\frac{2.2}{2+2-2}\right]^3 + 15\left[\frac{2.6}{2+6-2}\right]^3 + 97\left[\frac{2.4}{2+4-2}\right]^3 + 41\left[\frac{4.4}{4+4-2}\right]^3 \\ &+ 9\left[\frac{4.6}{4+6-2}\right]^3 \\ &= 1964.481481. \\ SDD[G] &= 6\left[\frac{2^2+2^2}{2.2}\right] + 15\left[\frac{2^2+6^2}{2.6}\right] + 97\left[\frac{2^2+4^2}{2.4}\right] + 41\left[\frac{4^2+4^2}{4.4}\right] \end{aligned}$$

$$+ \quad 9\left[\frac{4^2+6^2}{4.6}\right]$$
$$= \quad 406.$$

□

Theorem 20. *Let G be semi-total point graph R(G) of triglyceride. Then*

$$H[G] = 51.13333333.$$
$$SCI[G] = 65.24512394.$$
$$MD[G] = 272.$$

Proof. The graph G be semi-total point graph R(G) of triglyceride which consists of 5 different type of edge sets from Table 5.3. Using from the Table 5.3 we computed respective index as follows.

$$\therefore H[G] = 6\left[\frac{2}{2+2}\right]+15\left[\frac{2}{2+6}\right]+97\left[\frac{2}{2+4}\right]+41\left[\frac{2}{4+4}\right]+9\left[\frac{2}{4+6}\right]$$
$$= 51.13333333.$$
$$SCI[G] = 6\left[\frac{1}{\sqrt{2+2}}\right]+15\left[\frac{1}{\sqrt{2+6}}\right]+97\left[\frac{1}{\sqrt{2+4}}\right]+41\left[\frac{1}{\sqrt{4+4}}\right]$$
$$+ \quad 9\left[\frac{1}{\sqrt{4+6}}\right]$$

$$= 65.24512394.$$

$$MD[G] = 6|2-2| + 15|2-6| + 97|2-4| + 41|4-4| + 9|4-6|$$
$$= 272.$$

□

Theorem 21. *Let G be semi-total point graph R(G) of triglyceride. Then*

$$MRD[G] = 76.39583484.$$
$$MIRD[G] = 25.39800052.$$
$$MHD[G] = 22.125.$$

Proof. The graph G be semi-total point graph R(G) of triglyceride which consists of 5 different type of edge sets from Table 5.3. Using from the Table 5.3 we computed respective index as follows.

$$\therefore MRD[G] = 6|\sqrt{2}-\sqrt{2}| + 15|\sqrt{6}-\sqrt{2}| + 97|\sqrt{4}-\sqrt{2}| + 41|\sqrt{4}-\sqrt{4}|$$
$$+ 9|\sqrt{6}-\sqrt{4}|$$
$$= 76.39583484.$$

$$\begin{aligned}
MIRD[G] &= 6\left|\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right| + 15\left|\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\right| + 97\left|\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{4}}\right| + 41\left|\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{4}}\right| \\
&+ 9\left|\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{6}}\right| \\
&= 25.39800052.
\end{aligned}$$

$$\begin{aligned}
MHD[G] &= 6|2^{-2} - 2^{-2}| + 15|2^{-2} - 2^{-6}| + 97|2^{-2} - 2^{-4}| + 41|2^{-4} - 2^{-4}| \\
&+ 9|2^{-4} - 2^{-6}| \\
&= 22.125.
\end{aligned}$$

□

5.5 Semi-total line graph of triglyceride

Table 5.4: The edge partition of additional subdivision graph $Q(G)$ of triglyceride.

(d_u, d_v),$uv \in E(G)$	Number of edges
$(1,3)$	3
$(1,4)$	3
$(2,3)$	3
$(2,4)$	84
$(2,5)$	7
$(3,5)$	7
$(3,4)$	8

(d_u, d_v), $uv \in E(G)$	Number of edges
$(4,4)$	38
$(4,5)$	13
$(5,5)$	4

Theorem 22. *Let G be a semi-total point graph Q(G) of triglyceride. Then*

$$\begin{aligned} ISI[G] &= 279.4781746. \\ AZI[G] &= 2135.073013. \\ SDD[G] &= 402.7333333. \end{aligned}$$

Proof. The graph G be semi-total point graph Q(G) of triglyceride which consists of 10 different type of edge sets from Table 5.4. Using from the Table 5.4 we computed respective index as follows.

$$\begin{aligned} \therefore ISI[G] &= 3\left[\frac{1.3}{1+3}\right] + 3\left[\frac{1.4}{1+4}\right] + 3\left[\frac{2.3}{2+3}\right] + 84\left[\frac{2.4}{2+4}\right] \\ &+ 7\left[\frac{2.5}{2+5}\right] + 7\left[\frac{3.5}{3+5}\right] + 8\left[\frac{3.4}{3+4}\right] + 38\left[\frac{4.4}{4+4}\right] \\ &+ 13\left[\frac{4.5}{4+5}\right] + 4\left[\frac{5.5}{5+5}\right] \\ &= 279.4781746. \\ AZI[G] &= 3\left[\frac{1.3}{1+3-2}\right]^3 + 3\left[\frac{1.4}{1+4-2}\right]^3 + 3\left[\frac{2.3}{2+3-2}\right]^3 + 84\left[\frac{2.4}{2+4-2}\right]^3 \end{aligned}$$

$$
\begin{aligned}
&+ \; 7\left[\frac{2.5}{2+5-2}\right]^3 + 7\left[\frac{3.5}{3+5-2}\right]^3 + 8\left[\frac{3.4}{3+4-2}\right]^3 + 38\left[\frac{4.4}{4+4-2}\right]^3 \\
&+ \; 13\left[\frac{4.5}{4+5-2}\right]^3 + 4\left[\frac{5.5}{5+5-2}\right]^3 \\
&= \; 2135.073013.
\end{aligned}
$$

$$
\begin{aligned}
SDD[G] &= 3\left[\frac{1^2+3^2}{1.3}\right] + 3\left[\frac{1^2+4^2}{1.4}\right] + 3\left[\frac{2^2+3^2}{2.3}\right] + 84\left[\frac{2^2+4^2}{2.4}\right] \\
&+ 7\left[\frac{2^2+5^2}{2.5}\right] + 7\left[\frac{3^2+5^2}{3.5}\right] + 8\left[\frac{3^2+4^2}{3.4}\right] + 38\left[\frac{4^2+4^2}{4.4}\right] \\
&+ 13\left[\frac{4^2+5^2}{4.5}\right] + 4\left[\frac{5^2+5^2}{5.5}\right] \\
&= 402.733333.
\end{aligned}
$$

□

Theorem 23. *Let G be semi-total point graph Q(G) of triglyceride. Then*

$$
\begin{aligned}
H[G] &= 51.12460317. \\
SCI[G] &= 65.65375204. \\
MD[G] &= 242.
\end{aligned}
$$

Proof. The graph G be semi-total point graph Q(G) of triglyceride which consists of 10 different type of edge sets from Table 5.4. Using from the Table 5.4 we computed respective index as follows.

$$\begin{aligned}
\therefore H[G] &= 3\left[\frac{2}{1+3}\right]+3\left[\frac{2}{1+4}\right]+3\left[\frac{2}{2+3}\right]+84\left[\frac{2}{2+4}\right]+7\left[\frac{2}{2+5}\right]\\
&+ 7\left[\frac{2}{3+5}\right]+8\left[\frac{2}{3+4}\right]+38\left[\frac{2}{4+4}\right]+13\left[\frac{2}{4+5}\right]+4\left[\frac{2}{5+5}\right]\\
&= 51.12460317.\\
SCI[G] &= 3\left[\frac{1}{\sqrt{1+3}}\right]+3\left[\frac{1}{\sqrt{1+4}}\right]+3\left[\frac{1}{\sqrt{2+3}}\right]+84\left[\frac{1}{\sqrt{2+4}}\right]+7\left[\frac{1}{\sqrt{2+5}}\right]\\
&+ 7\left[\frac{1}{\sqrt{3+5}}\right]+8\left[\frac{1}{\sqrt{3+4}}\right]+38\left[\frac{1}{\sqrt{4+4}}\right]+13\left[\frac{1}{\sqrt{4+5}}\right]+4\left[\frac{1}{\sqrt{5+5}}\right]\\
&= 65.65375204.\\
MD[G] &= 3|1-3|+3|1-4|+3|2-3|+84|2-4|+7|2-5|+7|3-5|\\
&+ 8|3-4|+38|4-4|+13|4-5|+4|5-5|\\
&= 242.
\end{aligned}$$

□

Theorem 24. *Let G be semi-total point graph Q(G) of triglyceride. Then*

$$\begin{aligned}
MRD[G] &= 69.84930326.\\
MIRD[G] &= 24.58942278.
\end{aligned}$$

$$MHD[G] \;=\; 21.65625.$$

Proof. The graph G be semi-total point graph Q(G) of triglyceride which consists of 10 different type of edge sets from Table 5.4. Using from the Table 5.4 we computed respective index as follows.

$$\begin{aligned}
\therefore MRD[G] &= 3|\sqrt{1}-\sqrt{3}| + 3|\sqrt{1}-\sqrt{4}| + 3|\sqrt{2}-\sqrt{3}| + 84|\sqrt{2}-\sqrt{4}| \\
&+ 7|\sqrt{2}-\sqrt{5}| + 7|\sqrt{3}-\sqrt{5}| + 8|\sqrt{3}-\sqrt{4}| + 38|\sqrt{4}-\sqrt{4}| \\
&+ 13|\sqrt{4}-\sqrt{5}| + 4|\sqrt{5}-\sqrt{5}| \\
&= 69.84930326. \\
MIRD[G] &= 3\left|\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{3}}\right| + 3\left|\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{4}}\right| + 3\left|\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right| + 84\left|\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{4}}\right| \\
&+ 7\left|\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{5}}\right| + 7\left|\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{5}}\right| + 8\left|\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}\right| + 38\left|\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{4}}\right| \\
&+ 13\left|\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{5}}\right| + 4\left|\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{5}}\right| \\
&= 24.58942278 \\
MHD[G] &= 3|2^{-1}-2^{-3}| + 3|2^{-1}-2^{-4}| + 3|2^{-2}-2^{-3}| + 84|2^{-2}-2^{-4}| \\
&+ 7|2^{-2}-2^{-5}| + 7|2^{-3}-2^{-5}| + 8|2^{-3}-2^{-4}| + 13|2^{-4}-2^{-5}| \\
&+ 38|2^{-4}-2^{-4}| + 4|2^{-5}-2^{-5}| \\
&= 21.65625.
\end{aligned}$$

□

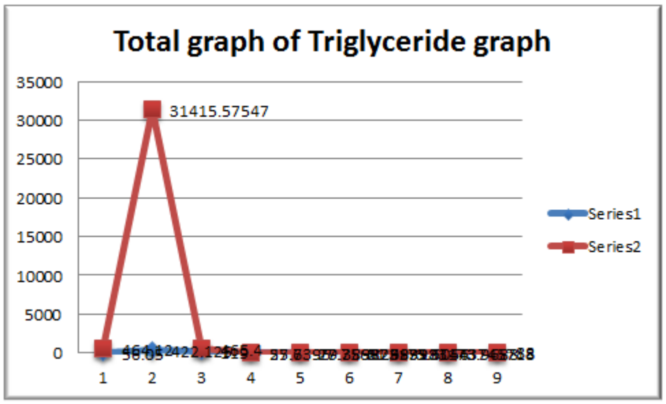

Figure 5.2: Comparison of general and total graph of triglyceride

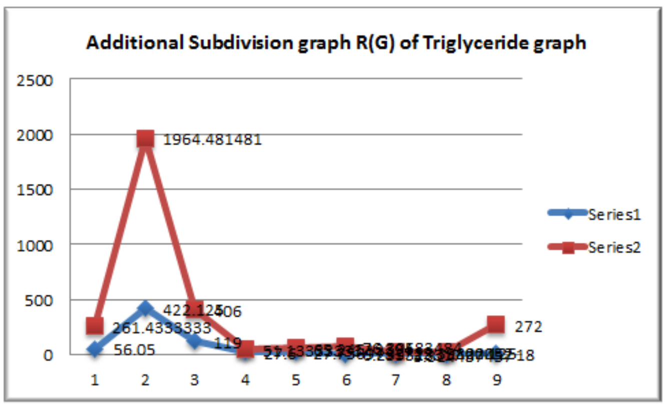

Figure 5.3: Comparison of general and semi-total point graph of triglyceride

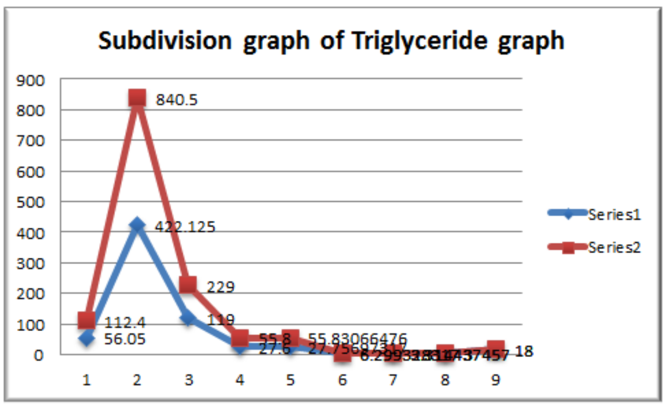

Figure 5.4: Comparison of general and subdivision graph of triglyceride

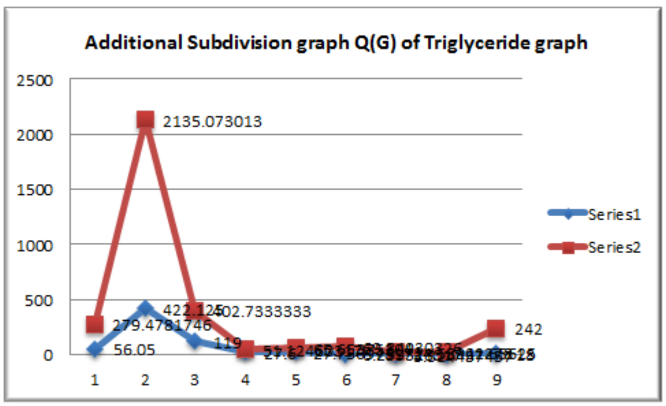

Figure 5.5: Comparison of general and semi-total line graph of triglyceride

Chapter 6

Investigation on splice graphs by exploiting certain topological indices

6.1 Preliminaries

Let G and H be two simple connected graphs with disjoint vertex sets $V(G)$ and $V(H)$, and edge sets $E(G)$ and $E(H)$ respectively. Let $b_1 \in V(G)$ and $y_1 \in V(H)$. Then the *splice graph* $G \bullet H$ of G and H by vertices b_1 and y_1 respectively, is defined by identifying the vertices b_1

and y_1 in the union of G and H (see, for instance, [3,13,41]). It is known that, for splice graphs, the total number of vertices is $n_G + n_H - 1$ while the total number of edges is $e_G + e_H$ (see below Figure 6.1).

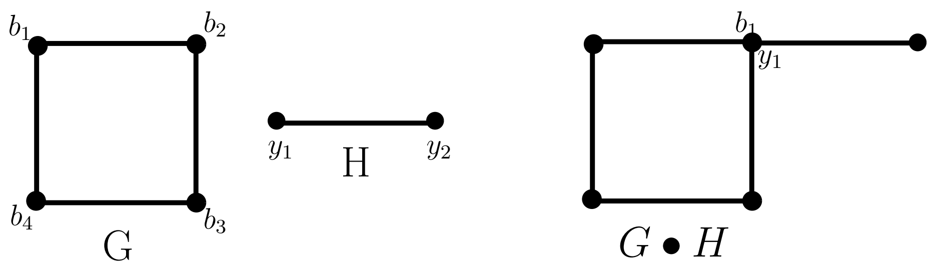

Figure 6.1: Splice of G and H by the vertices b_1 and y_1

6.2 Subdivision-vertex splice graph

Let G and H be two vertex disjoint graphs, and let $b_1 \in V(G)$ and $y_1 \in V(H)$. The subdivision vertex splice G and H is denoted by $G \bullet_v H$ and obtained from $S(G)$ and one copy of H which is identifying the vertices b_1 and y_1 in the union of $S(G)$ and H (see below Figure 6.2).

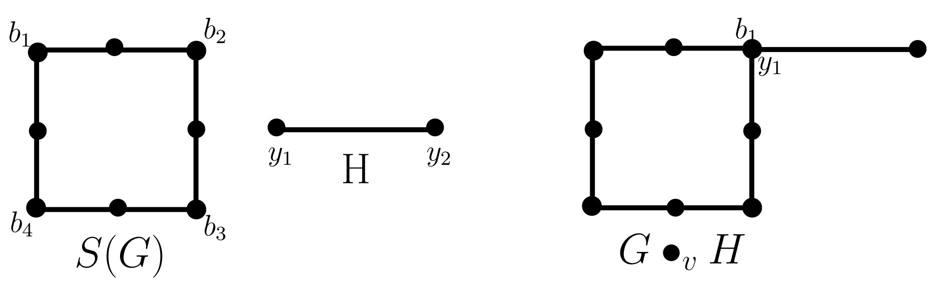

Figure 6.2: Subdivision-vertex splice

Theorem 25. *Let G and H are two simple connected graphs, then the bounds for the inverse sum indeg index of $G \bullet_v H$ are given by*

$$ISI[G \bullet_v H] \leq \frac{2\Delta_G[2m_1 - \Delta_G]}{\Delta_G + 2} + \frac{\Delta_H[m_2 - \Delta_H]}{2} + \frac{2\Delta_G(\Delta_G + \Delta_H)}{\Delta_G + \Delta_H + 2} + \frac{\Delta_H^2(\Delta_G + \Delta_H)}{\Delta_G + 2\Delta_H}.$$

$$ISI[G \bullet_v H] \geq \frac{2\delta_G[2m_1 - \delta_G]}{\delta_G + 2} + \frac{\delta_H[m_2 - \delta_H]}{2} + \frac{2\delta_G(\delta_G + \delta_H)}{\delta_G + \delta_H + 2} + \frac{\delta_H^2(\delta_G + \delta_H)}{\delta_G + 2\Delta_H}.$$

Proof. Consider,

$$\begin{aligned} ISI[G \bullet_v H] &= \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in V[G], v \in I[G]}} \left[\frac{d_G(u).2}{d_G(u) + 2}\right] + \sum_{\substack{uv \in E[G \bullet_v H] \\ u,v \in V[H]}} \left[\frac{d_H(u).d_H(v)}{d_H(u) + d_H(v)}\right] \\ &+ \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in M[G \bullet_v H], v \in I[G]}} \left[\frac{(d_G(u) + d_H(v)).2}{d_G(u) + d_H(v) + 2}\right] \\ &+ \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in M[G \bullet_v H], v \in V[H]}} \left[\frac{(d_G(u) + d_H(v)).d_H(w)}{d_G(u) + d_H(v) + d_H(w)}\right] \\ &= [2m_1 - d_G(S(u))]\left[\frac{2d_G(u)}{d_G(u) + 2}\right] + [m_2 - d_H(S(u))]\left[\frac{d_H(u).d_H(v)}{d_H(u) + d_H(v)}\right] \\ &+ d_G(S(u))\left[\frac{2(d_G(u) + d_H(v))}{d_G(u) + d_H(v) + 2}\right] \\ &+ d_H(S(u))\left[\frac{(d_G(u) + d_H(v)).d_H(w)}{d_G(u) + d_H(v) + d_H(w)}\right] \\ &\leq [2m_1 - \Delta_G]\left[\frac{2\Delta_G}{\Delta_G + 2}\right] + [m_2 - \Delta_H]\left[\frac{\Delta_H^2}{2\Delta_H}\right] + \Delta_G\left[\frac{2(\Delta_G + \Delta_H)}{\Delta_G + \Delta_H + 2}\right] \end{aligned}$$

$$+ \Delta_H \left[\frac{(\Delta_G + \Delta_H).\Delta_H}{\Delta_G + \Delta_H + \Delta_H} \right]$$

$$ISI[G \bullet_v H] \leq \frac{2\Delta_G[2m_1 - \Delta_G]}{\Delta_G + 2} + \frac{\Delta_H[m_2 - \Delta_H]}{2} + \frac{2\Delta_G(\Delta_G + \Delta_H)}{\Delta_G + \Delta_H + 2} + \frac{\Delta_H^2(\Delta_G + \Delta_H)}{\Delta_G + 2\Delta_H}.$$

One can analogously compute the following

$$ISI[G \bullet_v H] \geq \frac{2\delta_G[2m_1 - \delta_G]}{\delta_G + 2} + \frac{\delta_H[m_2 - \delta_H]}{2} + \frac{2\delta_G(\delta_G + \delta_H)}{\delta_G + \delta_H + 2} + \frac{\delta_H^2(\delta_G + \delta_H)}{\delta_G + 2\Delta_H}.$$

□

Theorem 26. *Let G and H are two simple connected graphs, then the bounds for the EM_1 index of $G \bullet_v H$ are given by*

$$EM_1[G \bullet_v H] \leq \Delta_G^2[2m_1 - \Delta_G] + 4[m_2 - \Delta_H][\Delta_H - 1]^2 + \Delta_G[\Delta_G + \Delta_H]^2 + \Delta_H[\Delta_G + 2\Delta_H - 2]^2.$$

$$EM_1[G \bullet_v H] \geq \delta_G^2[2m_1 - \delta_G] + 4[m_2 - \delta_H][\delta_H - 1]^2 + \delta_G[\delta_G + \delta_H]^2 + \delta_H[\delta_G + 2\delta_H - 2]^2.$$

Proof. Consider,

$$EM_1[G \bullet_v H] = \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in V[G], v \in I[G]}} [d_G(u) + 2 - 2]^2 + \sum_{\substack{uv \in E[G \bullet_v H] \\ u,v \in V[H]}} [d_H(u) + d_H(v) - 2]^2$$

$$+ \sum_{\substack{uv\in E[G\bullet_v H]\\ u\in M[G\bullet_v H], v\in I[G]}} [d_G(u) + d_H(v)) + 2 - 2]^2$$

$$+ \sum_{\substack{uv\in E[G\bullet_v H]\\ u\in M[G\bullet_v H], v\in V[H]}} [d_G(u) + d_H(v) + d_H(w) - 2]^2$$

$$= [2m_1 - d_G(S(u))][d_G(u)]^2 + [m_2 - d_H(S(u))][d_H(u) + d_H(v) - 2]^2$$

$$+ d_H(S(u))[d_G(u) + d_H(v) + d_H(w) - 2]^2$$

$$+ d_G(S(u))[d_G(u) + d_H(v)]^2$$

$$\leq \Delta_G^2[2m_1 - \Delta_G] + [m_2 - \Delta_H][\Delta_H + \Delta_H - 2]^2 + \Delta_G[\Delta_G + \Delta_H]^2$$

$$+ \Delta_H[\Delta_G + \Delta_H + \Delta_H - 2]^2$$

$$EM_1[G \bullet_v H] \leq \Delta_G^2[2m_1 - \Delta_G] + 4[m_2 - \Delta_H][\Delta_H - 1]^2 + \Delta_G[\Delta_G + \Delta_H]^2$$

$$+ \Delta_H[\Delta_G + 2\Delta_H - 2]^2.$$

One can analogously compute the following

$$EM_1[G \bullet_v H] \geq \delta_G^2[2m_1 - \delta_G] + 4[m_2 - \delta_H][\delta_H - 1]^2 + \delta_G[\delta_G + \delta_H]^2$$

$$+ \delta_H[\delta_G + 2\delta_H - 2]^2.$$

□

Theorem 27. *Let G and H are two simple connected graphs, then the bounds for the atom-bond connectivite index of $G \bullet_v H$ are given by*

$$ABC[G \bullet_v H] \leq \frac{[2m_1 - \Delta_G]}{\sqrt{2}} + [m_2 - \Delta_H]\sqrt{\frac{2(\Delta_H - 1)}{\Delta_H^2}} + \frac{\Delta_G}{\sqrt{2}} + \Delta_H\sqrt{\frac{\Delta_G + 2\Delta_H - 2}{(\Delta_G + \Delta_H).\Delta_H}}$$

$$ABC[G \bullet_v H] \geq \frac{[2m_1 - \delta_G]}{\sqrt{2}} + [m_2 - \delta_H]\sqrt{\frac{2(\delta_H - 1)}{\delta_H^2}} + \frac{\delta_G}{\sqrt{2}} + \delta_H\sqrt{\frac{\delta_G + 2\delta_H - 2}{(\delta_G + \delta_H).\delta_H}}$$

Proof. Consider,

$$\begin{aligned} ABC[G \bullet_v H] = & \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in V[G], v \in I[G]}} \left[\sqrt{\frac{d_G(u) + 2 - 2}{d_G(u).2}}\right] \\ & + \sum_{\substack{uv \in E[G \bullet_v H] \\ u,v \in V[H]}} \left[\sqrt{\frac{d_H(u) + d_H(v) - 2}{d_H(u).d_H(v)}}\right] \\ & + \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in M[G \bullet_v H], v \in I[G]}} \left[\sqrt{\frac{(d_G(u) + d_H(v)) + 2 - 2}{(d_G(u) + d_H(v)).2}}\right] \\ & + \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in M[G \bullet_v H], v \in V[H]}} \left[\sqrt{\frac{(d_G(u) + d_H(v)) + d_H(w) - 2}{(d_G(u) + d_H(v)).d_H(w)}}\right] \end{aligned}$$

$$= [2m_1 - d_G(S(u))]\left[\sqrt{\frac{d_G(u)}{2d_G(u)}}\right]$$

$$+ [m_2 - d_H(S(u))]\left[\sqrt{\frac{d_H(u) + d_H(v) - 2}{d_H(u).d_H(v)}}\right]$$

$$+ d_G(S(u))\left[\sqrt{\frac{(d_G(u) + d_H(v))}{2(d_G(u) + d_H(v))}}\right]$$

$$+ d_H(S(u))\left[\sqrt{\frac{(d_G(u) + d_H(v)) + d_H(w) - 2}{(d_G(u) + d_H(v)).d_H(w)}}\right]$$

$$\leq \frac{[2m_1 - \Delta_G]}{\sqrt{2}} + [m_2 - \Delta_H]\sqrt{\frac{\Delta_H + \Delta_H - 2}{\Delta_H.\Delta_H}} + \frac{\Delta_G}{\sqrt{2}}$$

$$+ \Delta_H\sqrt{\frac{\Delta_G + \Delta_H + \Delta_H - 2}{(\Delta_G + \Delta_H).\Delta_H}}$$

$$ABC[G \bullet_v H] \leq \frac{[2m_1 - \Delta_G]}{\sqrt{2}} + [m_2 - \Delta_H]\sqrt{\frac{2(\Delta_H - 1)}{\Delta_H^2}} + \frac{\Delta_G}{\sqrt{2}}$$

$$+ \Delta_H\sqrt{\frac{\Delta_G + 2\Delta_H - 2}{(\Delta_G + \Delta_H).\Delta_H}}.$$

One can analogously compute the following

$$ABC[G \bullet_v H] \geq \frac{[2m_1 - \delta_G]}{\sqrt{2}} + [m_2 - \delta_H]\sqrt{\frac{2(\delta_H - 1)}{\delta_H^2}} + \frac{\delta_G}{\sqrt{2}}$$

$$+ \delta_H\sqrt{\frac{\delta_G + 2\delta_H - 2}{(\delta_G + \delta_H).\delta_H}}.$$

□

Theorem 28. *Let G and H are two simple connected graphs, then the bounds for the SK_1 index of $G \bullet_v H$ are given by*

$$SK_1[G \bullet_v H] \leq [2m_1 - \Delta_G]\left[\frac{\Delta_G + 2}{2}\right] + [m_2 - \Delta_H]\Delta_H + \Delta_G\left[\frac{\Delta_G + \Delta_H + 2}{2}\right] + \Delta_H\left[\frac{\Delta_G + 2\Delta_H}{2}\right].$$

$$SK_1[G \bullet_v H] \geq [2m_1 - \delta_G]\left[\frac{\delta_G + 2}{2}\right] + [m_2 - \delta_H]\delta_H + \delta_G\left[\frac{\delta_G + \delta_H + 2}{2}\right] + \delta_H\left[\frac{\delta_G + 2\delta_H}{2}\right].$$

Proof. Consider,

$$\begin{aligned}
SK_1[G \bullet_v H] &= \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in V[G], v \in I[G]}} \left[\frac{d_G(u) + 2}{2}\right] + \sum_{\substack{uv \in E[G \bullet_v H] \\ u, v \in V[H]}} \left[\frac{d_H(u) + d_H(v)}{2}\right] \\
&+ \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in M[G \bullet_v H], v \in I[G]}} \left[\frac{(d_G(u) + d_H(v)) + 2}{2}\right] \\
&+ \sum_{\substack{uv \in E[G \bullet_v H] \\ u \in M[G \bullet_v H], v \in V[H]}} \left[\frac{(d_G(u) + d_H(v)) + d_H(w)}{2}\right] \\
&= [2m_1 - d_G(S(u))]\left[\frac{d_G(u) + 2}{2}\right] + [m_2 - d_H(S(u))]\left[\frac{d_H(u) + d_H(v)}{2}\right] \\
&+ d_G(S(u))\left[\frac{d_G(u) + d_H(v) + 2}{2}\right] \\
&+ d_H(S(u))\left[\frac{d_G(u) + d_H(v) + d_H(w)}{2}\right]
\end{aligned}$$

$$\leq [2m_1 - \Delta_G]\left[\frac{\Delta_G + 2}{2}\right] + [m_2 - \Delta_H]\left[\frac{\Delta_H + \Delta_H}{2}\right]$$
$$+ \Delta_G\left[\frac{\Delta_G + \Delta_H + 2}{2}\right] + \Delta_H\left[\frac{\Delta_G + \Delta_H + \Delta_H}{2}\right]$$
$$\leq [2m_1 - \Delta_G]\left[\frac{\Delta_G + 2}{2}\right] + [m_2 - \Delta_H]\left[\frac{2\Delta_H}{2}\right]$$
$$+ \Delta_G\left[\frac{\Delta_G + \Delta_H + 2}{2}\right] + \Delta_H\left[\frac{\Delta_G + 2\Delta_H}{2}\right]$$
$$SK_1[G \bullet_v H] \leq [2m_1 - \Delta_G]\left[\frac{\Delta_G + 2}{2}\right] + [m_2 - \Delta_H]\Delta_H + \Delta_G\left[\frac{\Delta_G + \Delta_H + 2}{2}\right]$$
$$+ \Delta_H\left[\frac{\Delta_G + 2\Delta_H}{2}\right].$$

One can analogously compute the following

$$SK_1[G \bullet_v H] \geq [2m_1 - \delta_G]\left[\frac{\delta_G + 2}{2}\right] + [m_2 - \delta_H]\delta_H + \delta_G\left[\frac{\delta_G + \delta_H + 2}{2}\right]$$
$$+ \delta_H\left[\frac{\delta_G + 2\delta_H}{2}\right].$$

□

6.3 Subdivision-edge splice graph

Let $p_2 \in I(G)$ be the inserted vertex of $S(G)$, and let $y_1 \in V(H)$. Then the S-edge splice of G and H is denoted by $G \bullet_e H$ that is obtained from $S(G)$ and one copy of H identifying the vertices p_2 and y_1 in the union of $S(G)$ and H (see below Figure 6.3).

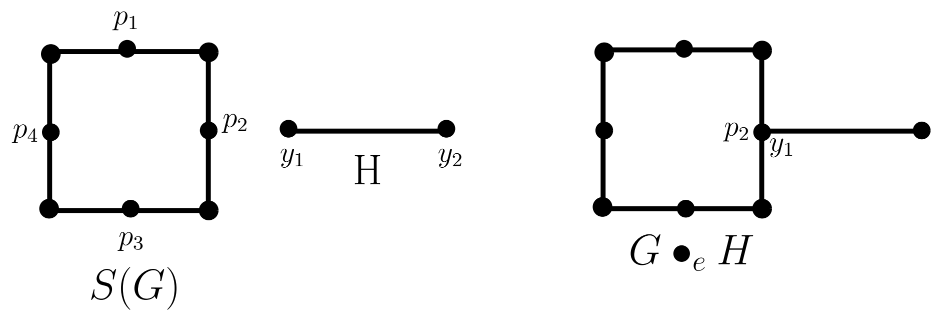

Figure 6.3: Subdivision-edge splice graph

Theorem 29. *Let G and H are two simple connected graphs, then the bounds for the inverse sum indeg index of $G \bullet_e H$ are given by*

$$ISI[G \bullet_e H] \leq \frac{4\Delta_G[m_1 - 1]}{\Delta_G + 2} + \frac{\Delta_H[m_2 - \Delta_H]}{2} + 2\left[\frac{\Delta_G(\Delta_H + 2)}{\Delta_G + \Delta_H + 2}\right] + \left[\frac{\Delta_H^2(\Delta_H + 2)}{2(\Delta_H + 1)}\right].$$

$$ISI[G \bullet_e H] \leq \frac{4\delta_G[m_1 - 1]}{\delta_G + 2} + \frac{\delta_H[m_2 - \delta_H]}{2} + 2\left[\frac{\delta_G(\delta_H + 2)}{\delta_G + \delta_H + 2}\right] + \left[\frac{\delta_H^2(\delta_H + 2)}{2(\delta_H + 1)}\right].$$

Proof. Consider,

$$ISI[G \bullet_e H] = \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in V[G], v \in I[G]}} \left[\frac{d_G(u).2}{d_G(u) + 2}\right] + \sum_{\substack{uv \in E[G \bullet_e H] \\ u,v \in V[H]}} \left[\frac{d_H(u).d_H(v)}{d_H(u) + d_H(v)}\right] + \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in M[G \bullet_e H], v \in V[G]}} \left[\frac{(d_H(v) + 2).d_G(u)}{(d_H(v) + 2) + d_G(u)}\right]$$

$$+ \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in M[G \bullet_e H], v \in V[H]}} \left[\frac{(d_H(u)+2).d_H(v)}{(d_H(v)+2)+d_H(v)} \right]$$

$$= [2m_1 - 2]\left[\frac{d_G(u).2}{d_G(u)+2}\right] + [m_2 - d_H(S(u))]\left[\frac{d_H(u).d_H(v)}{d_H(u)+d_H(v)}\right]$$

$$+ 2\left[\frac{(d_H(v)+2).d_G(u)}{(d_H(v)+2)+d_G(u)}\right] + d_H(S(u))\left[\frac{(d_H(u)+2).d_H(v)}{(d_H(v)+2)+d_H(v)}\right]$$

$$\leq [2m_1 - 2]\left[\frac{2\Delta_G}{\Delta_G+2}\right] + [m_2 - \Delta_H]\left[\frac{\Delta_H.\Delta_H}{\Delta_H+\Delta_H}\right]$$

$$+ 2\left[\frac{(\Delta_H+2).\Delta_G}{\Delta_H+\Delta_G+2}\right] + \Delta_H\left[\frac{\Delta_H(\Delta_H+2)}{\Delta_H+\Delta_H+2}\right]$$

$$\leq 2[m_1 - 1]\left[\frac{2\Delta_G}{\Delta_G+2}\right] + [m_2 - \Delta_H]\left[\frac{\Delta_H^2}{2\Delta_H}\right]$$

$$+ 2\left[\frac{(\Delta_H+2).\Delta_G}{\Delta_H+\Delta_G+2}\right] + \Delta_H\left[\frac{\Delta_H(\Delta_H+2)}{2\Delta_H+2}\right]$$

$$ISI[G \bullet_e H] \leq \frac{4\Delta_G[m_1-1]}{\Delta_G+2} + \frac{\Delta_H[m_2-\Delta_H]}{2} + 2\left[\frac{\Delta_G(\Delta_H+2)}{\Delta_G+\Delta_H+2}\right]$$

$$+ \left[\frac{\Delta_H^2(\Delta_H+2)}{2(\Delta_H+1)}\right].$$

One can analogously compute the following

$$ISI[G \bullet_e H] \leq \frac{4\delta_G[m_1-1]}{\delta_G+2} + \frac{\delta_H[m_2-\delta_H]}{2} + 2\left[\frac{\delta_G(\delta_H+2)}{\delta_G+\delta_H+2}\right] + \left[\frac{\delta_H^2(\delta_H+2)}{2(\delta_H+1)}\right].$$

□

Theorem 30. *Let G and H are two simple connected graphs, then the bounds for the EM_1 index of $G \bullet_e H$ are given by*

$$EM_1[G \bullet_e H] \leq 2\Delta_G^2[m_1 - 1] + 4[m_2 - \Delta_H][\Delta_H - 1]^2 + 2[\Delta_G + \Delta_H]^2 + 4\Delta_H^3.$$

$$EM_1[G \bullet_e H] \geq 2\delta_G^2[m_1 - 1] + 4[m_2 - \delta_H][\delta_H - 1]^2 + 2[\delta_G + \delta_H]^2 + 4\delta_H^3.$$

Proof. Consider,

$$EM_1[G \bullet_e H] = \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in V[G], v \in I[G]}} [d_G(u) + 2 - 2]^2 + \sum_{\substack{uv \in E[G \bullet_e H] \\ u,v \in V[H]}} [d_H(u) + d_H(v) - 2]^2$$

$$+ \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in M[G \bullet_e H], v \in V[G]}} [(d_H(v) + 2) + d_G(v) - 2]^2$$

$$+ \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in M[G \bullet_e H], v \in V[H]}} [(d_H(u) + 2) + d_H(v) - 2]^2$$

$$= [2m_1 - 2][d_G(u)]^2 + [m_2 - d_H(S(u))][d_H(u) + d_H(v) - 2]^2$$

$$+ 2[d_H(u) + 2 + d_G(v) - 2]^2 + d_H(S(u))[d_H(u) + 2 + d_H(v) - 2]^2$$

$$\leq 2\Delta_G^2[m_1 - 1] + [m_2 - \Delta_H][\Delta_H + \Delta_H - 2]^2 + 2[\Delta_G + \Delta_H]^2$$

$$+ \Delta_H[\Delta_H + \Delta_H]^2$$

$$\leq 2\Delta_G^2[m_1 - 1] + [m_2 - \Delta_H][2\Delta_H - 2]^2 + 2[\Delta_G + \Delta_H]^2 + \Delta_H[2\Delta_H]^2$$

$$EM_1[G \bullet_e H] \leq 2\Delta_G^2[m_1 - 1] + 4[m_2 - \Delta_H][\Delta_H - 1]^2 + 2[\Delta_G + \Delta_H]^2 + 4\Delta_H^3.$$

One can analogously compute the following

$$EM_1[G \bullet_e H] \geq 2\delta_G^2[m_1-1] + 4[m_2-\delta_H][\delta_H-1]^2 + 2[\delta_G+\delta_H]^2 + 4\delta_H^3.$$

□

Theorem 31. *Let G and H are two simple connected graphs, then the bounds for the atom-bond connectivite index of $G \bullet_e H$ are given by*

$$ABC[G \bullet_e H] \leq \sqrt{2}[m_1-1] + [m_2-\Delta_H]\frac{\sqrt{2(\Delta_H-1)}}{\Delta_H} + 2\sqrt{\frac{\Delta_H+\Delta_G}{\Delta_G.(\Delta_H+2)}} + \Delta_H\sqrt{\frac{2}{\Delta_H+2}}.$$

$$ABC[G \bullet_e H] \geq \sqrt{2}[m_1-1] + [m_2-\delta_H]\frac{\sqrt{2(\delta_H-1)}}{\delta_H} + 2\sqrt{\frac{\delta_H+\delta_G}{\delta_G.(\delta_H+2)}} + \delta_H\sqrt{\frac{2}{\delta_H+2}}.$$

Proof. Consider,

$$ABC[G \bullet_e H] = \sum_{\substack{uv\in E[G\bullet_e H]\\ u\in V[G], v\in I[G]}} \left[\sqrt{\frac{d_G(u)+2-2}{d_G(u).2}}\right] + \sum_{\substack{uv\in E[G\bullet_e H]\\ u,v\in V[H]}} \left[\sqrt{\frac{d_H(u)+d_H(v)-2}{d_H(u).d_H(v)}}\right] + \sum_{\substack{uv\in E[G\bullet_e H]\\ u\in M[G\bullet_e H], v\in V[G]}} \left[\sqrt{\frac{(d_H(u)+2)+d_G(v)-2}{(d_H(u)+2).d_G(v)}}\right]$$

$$+ \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in M[G \bullet_e H], v \in V[H]}} \left[\sqrt{\frac{(d_H(u)+2)+d_H(v)-2}{(d_H(u)+2).d_H(v)}} \right].$$

$$= [2m_1 - 2]\left[\sqrt{\frac{d_G(u)}{2d_G(u)}}\right] + [m_2 - d_H(S(u))]\left[\sqrt{\frac{d_H(u)+d_H(v)-2}{d_H(u).d_H(v)}}\right]$$

$$+ 2\left[\sqrt{\frac{d_H(u)+2+d_G(v)-2}{d_G(u).(d_H(v)+2)}}\right]$$

$$+ d_H(S(u))\left[\sqrt{\frac{d_H(u)+2+d_H(v)-2}{d_H(v)(d_H(u)+2)}}\right]$$

$$\leq \frac{2[m_1-1]}{\sqrt{2}} + [m_2 - \Delta_H]\sqrt{\frac{\Delta_H + \Delta_H - 2}{\Delta_H.\Delta_H}}$$

$$+ 2\sqrt{\frac{\Delta_H + \Delta_G}{\Delta_G.(\Delta_H + 2)}} + \Delta_H\sqrt{\frac{\Delta_H + \Delta_H}{\Delta_H.(\Delta_H + 2)}}$$

$$\leq \sqrt{2}[m_1 - 1] + [m_2 - \Delta_H]\sqrt{\frac{(2\Delta_H - 2)}{\Delta_H^2}}$$

$$+ 2\sqrt{\frac{\Delta_H + \Delta_G}{\Delta_G.(\Delta_H + 2)}} + \Delta_H\sqrt{\frac{2\Delta_H}{\Delta_H.(\Delta_H + 2)}}$$

$$ABC[G \bullet_e H] \leq \sqrt{2}[m_1 - 1] + [m_2 - \Delta_H]\frac{\sqrt{2(\Delta_H - 1)}}{\Delta_H}$$

$$+ 2\sqrt{\frac{\Delta_H + \Delta_G}{\Delta_G.(\Delta_H + 2)}} + \Delta_H\sqrt{\frac{2}{\Delta_H + 2}}.$$

One can analogously compute the following

$$ABC[G \bullet_e H] \geq \sqrt{2}[m_1 - 1] + [m_2 - \delta_H]\frac{\sqrt{2(\delta_H - 1)}}{\delta_H}$$

$$+ 2\sqrt{\frac{\delta_H + \delta_G}{\delta_G.(\delta_H + 2)}} + \delta_H\sqrt{\frac{2}{\delta_H + 2}}.$$

□

Theorem 32. *Let G and H are two simple connected graphs, then the bounds for the SK_1 index of $G \bullet_e H$ are given by*

$$SK_1[G \bullet_e H] \leq [m_1 - 1][\Delta_G + 2] + \Delta_H[m_2 - \Delta_H] + [\Delta_G + \Delta_H + 2] + \Delta_H[\Delta_H + 1].$$

$$SK_1[G \bullet_e H] \geq [m_1 - 1][\delta_G + 2] + \delta_H[m_2 - \delta_H] + [\delta_G + \delta_H + 2] + \delta_H[\delta_H + 1].$$

Proof. Consider,

$$\begin{aligned}
SK_1[G \bullet_e H] = & \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in V[G], v \in I[G]}} \left[\frac{d_G(u) + 2}{2}\right] + \sum_{\substack{uv \in E[G \bullet_e H] \\ u,v \in V[H]}} \left[\frac{d_H(u) + d_H(v)}{2}\right] \\
& + \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in M[G \bullet_e H], v \in V[G]}} \left[\frac{(d_H(u) + 2) + d_G(v)}{2}\right] \\
& + \sum_{\substack{uv \in E[G \bullet_e H] \\ u \in M[G \bullet_e H], v \in V[H]}} \left[\frac{(d_H(u) + 2) + d_H(v)}{2}\right] \\
& = [2m_1 - 2]\left[\frac{d_G(u) + 2}{2}\right] + [m_2 - d_H(S(u))]\left[\frac{2d_H(u)}{2}\right] \\
& + 2\left[\frac{d_H(u) + d_G(v) + 2}{2}\right] + d_H(S(u))\left[\frac{d_H(u) + 2 + d_H(v)}{2}\right]
\end{aligned}$$

$$\leq 2[m_1 - 1]\left[\frac{\Delta_G + 2}{2}\right] + [m_2 - \Delta_H]\Delta_H$$

$$+ 2\left[\frac{\Delta_G + \Delta_H + 2}{2}\right] + \Delta_H\left[\frac{\Delta_H + \Delta_H + 2}{2}\right]$$

$$\leq [m_1 - 1][\Delta_G + 2] + \Delta_H[m_2 - \Delta_H]$$

$$+ [\Delta_G + \Delta_H + 2] + \Delta_H\left[\frac{2(\Delta_H + 1)}{2}\right]$$

$$SK[G \bullet_e H] \leq [m_1 - 1][\Delta_G + 2] + \Delta_H[m_2 - \Delta_H] + [\Delta_G + \Delta_H + 2]$$

$$+ \Delta_H[\Delta_H + 1].$$

One can analogously compute the following

$$SK[G \bullet_e H] \geq [m_1 - 1][\delta_G + 2] + \delta_H[m_2 - \delta_H] + [\delta_G + \delta_H + 2] + \delta_H[\delta_H + 1].$$

□

6.4 Subdivision-vertex neighbourhood splice Graph

Let $b_1 \in V(G)$ and $y_1 \in V(H)$. The S-vertex neighbourhood splice of G and H is denoted by $G \bullet_{nv} H$ and obtained from $S(G)$ and $d(b_1)$ copies of

H and identifying the neighbourhood vertices of b_1. For $y_1 \in V(H)$, the union of the corresponding neighbourhood separated vertices $b_1 \in V(G)$ of $S(G)$ (see below Figure 6.4).

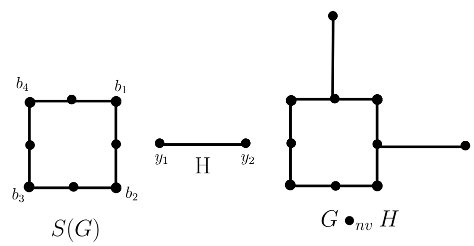

Figure 6.4: Subdivision- vertex neighbourhood splice

Theorem 33. *Let G and H are two simple connected graphs, then the bounds for the inverse sum indeg index of $G \bullet_{nv} H$ are given by*

$$ISI[G \bullet_{nv} H] \leq 2[m_1 - \Delta_G]\left[\frac{2\Delta_G}{\Delta_G + 2}\right] + \frac{\Delta_G \Delta_H}{2}[m_2 - \Delta_H] + 2\Delta_G\left[\frac{\Delta_G(2+\Delta_H)}{\Delta_G + \Delta_H + 2}\right] + \Delta_G\Delta_H\left[\frac{\Delta_H(2+\Delta_H)}{2(1+\Delta_H)}\right].$$

$$ISI[G \bullet_{nv} H] \geq 2[m_1 - \delta_G]\left[\frac{2\delta_G}{\delta_G + 2}\right] + \frac{\delta_G \delta_H}{2}[m_2 - \delta_H] + 2\delta_G\left[\frac{\delta_G(2+\delta_H)}{\delta_G + \delta_H + 2}\right] + \delta_G\delta_H\left[\frac{\delta_H(2+\delta_H)}{2(1+\delta_H)}\right].$$

Proof. Consider,

$$ISI[G \bullet_{nv} H] = \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in V[G], v \in I[G]}} \left[\frac{d_G(u).2}{d_G(u) + 2}\right] + \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u,v \in V[H]}} \left[\frac{d_H(u).d_H(v)}{d_H(u) + d_H(v)}\right]$$

$$
\begin{aligned}
& + \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in M[G \bullet_{nv} H], v \in V[G]}} \left[\frac{d_G(u).(2 + d_H(v))}{d_G(u)(2 + d_H(v))} \right] \\
& + \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in M[G \bullet_{nv} H], v \in V[H]}} \left[\frac{(2 + d_H(u)).d_H(v)}{(2 + d_H(u)) + d_H(v)} \right] \\
& = 2[m_1 - d_G(S(u))] \left[\frac{2d_G(u)}{d_G(u) + 2} \right] \\
& + d_G(S(u))[m_2 - d_H(S(u))] \left[\frac{d_H(u).d_H(v)}{d_H(u) + d_H(v)} \right] \\
& + 2d_G(S(u)) \left[\frac{d_G(u)(2 + d_H(v))}{d_G(u) + d_H(v) + 2} \right] \\
& + d_G(S(u))d_H(S(u)) \left[\frac{d_H(v)(2 + d_H(u))}{2 + d_H(u) + d_H(v)} \right] \\
& = 2[m_1 - d_G(S(u))] \left[\frac{2d_G(u)}{d_G(u) + 2} \right] \\
& + d_G(S(u))[m_2 - d_H(S(u))] \left[\frac{d_H(u)^2}{2d_H(u)} \right] \\
& + 2d_G(S(u)) \left[\frac{d_G(u)(2 + d_H(v))}{d_G(u) + d_H(v) + 2} \right] \\
& + d_G(S(u))d_H(S(u)) \left[\frac{d_H(v)(2 + d_H(u))}{2(1 + d_H(u))} \right] \\
ISI[G \bullet_{nv} H] & \leq 2[m_1 - \Delta_G] \left[\frac{2\Delta_G}{\Delta_G + 2} \right] + \frac{\Delta_G \Delta_H}{2}[m_2 - \Delta_H] \\
& + 2\Delta_G \left[\frac{\Delta_G(2 + \Delta_H)}{\Delta_G + \Delta_H + 2} \right] + \Delta_G \Delta_H \left[\frac{\Delta_H(2 + \Delta_H)}{2(1 + \Delta_H)} \right].
\end{aligned}
$$

One can analogously compute the following

$$ISI[G \bullet_{nv} H] \geq 2[m_1 - \delta_G]\left[\frac{2\delta_G}{\delta_G + 2}\right] + \frac{\delta_G\delta_H}{2}[m_2 - \delta_H] + 2\delta_G\left[\frac{\delta_G(2+\delta_H)}{\delta_G + \delta_H + 2}\right] + \delta_G\delta_H\left[\frac{\delta_H(2+\delta_H)}{2(1+\delta_H)}\right].$$

□

Theorem 34. *Let G and H are two simple connected graphs, then the bounds for the EM_1 index of $G \bullet_{nv} H$ are given by*

$$EM_1[G \bullet_{nv} H] \leq 2\Delta_G^2[m_1 - \Delta_G] + 4\Delta_G[m_2 - \Delta_H][\Delta_H - 1]^2 + 2\Delta_G[\Delta_G + \Delta_H]^2 + 4\Delta_G\Delta_H^3.$$

$$EM_1[G \bullet_{nv} H] \geq 2\delta_G^2[m_1 - \delta_G] + 4\delta_G[m_2 - \delta_H][\delta_H - 1]^2 + 2\delta_G[\delta_G + \delta_H]^2 + 4\delta_G\delta_H^3.$$

Proof. Consider,

$$EM_1[G \bullet_{nv} H] = \sum_{\substack{uv\in E[G\bullet_{nv}H] \\ u\in V[G],v\in I[G]}} [d_G(u) + 2 - 2]^2 + \sum_{\substack{uv\in E[G\bullet_{nv}H] \\ u,v\in V[H]}} [d_H(u) + d_H(v) - 2]^2 + \sum_{\substack{uv\in E[G\bullet_{nv}H] \\ u\in M[G\bullet_{nv}H],v\in V[G]}} [d_G(u) + d_H(v)) + 2 - 2]^2$$

$$+ \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in M[G \bullet_{nv} H], v \in V[H]}} [2 + d_H(u) + d_H(v) - 2]^2$$

$$= 2[m_1 - d_G(S(u))][d_G(u)]^2$$

$$+ d_G(S(u))[m_2 - d_H(S(u))][d_H(u) + d_H(v) - 2]^2$$

$$+ 2d_G(S(u))[d_G(u) + d_H(v)]^2$$

$$+ d_G(S(u))d_H(S(u))[d_H(u) + d_H(v)]^2$$

$$\leq 2\Delta_G^2[m_1 - \Delta_G] + \Delta_G[m_2 - \Delta_H][\Delta_H + \Delta_H - 2]^2$$

$$+ 2\Delta_G[\Delta_G + \Delta_H]^2 + \Delta_G\Delta_H[\Delta_H + \Delta_H]^2$$

$$EM_1[G \bullet_{nv} H] \leq 2\Delta_G^2[m_1 - \Delta_G] + 4\Delta_G[m_2 - \Delta_H][\Delta_H - 1]^2$$

$$+ 2\Delta_G[\Delta_G + \Delta_H]^2 + 4\Delta_G\Delta_H^3.$$

One can analogously compute the following

$$EM_1[G \bullet_{nv} H] \geq 2\delta_G^2[m_1 - \delta_G] + 4\delta_G[m_2 - \delta_H][\delta_H - 1]^2$$

$$+ 2\delta_G[\delta_G + \delta_H]^2 + 4\delta_G\delta_H^3.$$

□

Theorem 35. *Let G and H are two simple connected graphs, then the bounds for the atom-bond connectivite index of $G \bullet_{nv} H$ are given by*

$$ABC[G \bullet_{nv} H] \leq \sqrt{2}[m_1 - \Delta_G] + \frac{\Delta_G}{\Delta_H}[m_2 - \Delta_H]\sqrt{2(\Delta_H - 1)} + 2\Delta_G\sqrt{\frac{\Delta_G + \Delta_H}{\Delta_G(2 + \Delta_H)}} + \Delta_G\Delta_H\sqrt{\frac{2}{2 + \Delta_H}}$$

$$ABC[G \bullet_{nv} H] \geq \sqrt{2}[m_1 - \delta_G] + \frac{\delta_G}{\delta_H}[m_2 - \delta_H]\sqrt{2(\delta_H - 1)} + 2\delta_G\sqrt{\frac{\delta_G + \delta_H}{\delta_G(2 + \delta_H)}} + \delta_G\delta_H\sqrt{\frac{2}{2 + \delta_H}}.$$

Proof. Consider,

$$\begin{aligned}
ABC[G \bullet_{nv} H] = & \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in V[G], v \in I[G]}} \left[\sqrt{\frac{d_G(u) + 2 - 2}{d_G(u).2}}\right] \\
& + \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u,v \in V[H]}} \left[\sqrt{\frac{d_H(u) + d_H(v) - 2}{d_H(u).d_H(v)}}\right] \\
& + \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in M[G \bullet_{nv} H], v \in V[G]}} \left[\sqrt{\frac{d_G(u) + 2 + d_H(v) - 2}{d_G(u)(2 + d_H(v))}}\right] \\
& + \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in M[G \bullet_{nv} H], v \in V[H]}} \left[\sqrt{\frac{2 + d_H(u) + d_H(v) - 2}{(2 + d_H(u)).d_H(v)}}\right] \\
& = 2[m_1 - d_G(S(u))]\left[\sqrt{\frac{d_G(u)}{2d_G(u)}}\right]
\end{aligned}$$

$$
\begin{aligned}
&+ d_G(S(u))[m_2 - d_H(S(u))]\left[\sqrt{\frac{d_H(u)+d_H(v)-2}{d_H(u).d_H(v)}}\right] \\
&+ 2d_G(S(u))\left[\sqrt{\frac{d_G(u)+d_H(v)}{d_G(u)(2+d_H(v))}}\right] \\
&+ d_G(S(u))d_H(S(u))\left[\sqrt{\frac{d_H(u)+d_H(v)}{(2+d_H(u)).d_H(v)}}\right] \\
&\leq \sqrt{2}[m_1 - \Delta_G] + \Delta_G[m_2 - \Delta_H]\sqrt{\frac{\Delta_H+\Delta_H-2}{\Delta_H.\Delta_H}} \\
&+ 2\Delta_G\sqrt{\frac{\Delta_G+\Delta_H}{\Delta_G(2+\Delta_H)}} + \Delta_G\Delta_H\sqrt{\frac{\Delta_H+\Delta_H}{\Delta_H(2+\Delta_H)}} \\
&\leq \sqrt{2}[m_1 - \Delta_G] + \Delta_G[m_2 - \Delta_H]\sqrt{\frac{(2\Delta_H-2}{\Delta_H^2}} \\
&+ 2\Delta_G\sqrt{\frac{\Delta_G+\Delta_H}{\Delta_G(2+\Delta_H)}} + \Delta_G\Delta_H\sqrt{\frac{2\Delta_H}{\Delta_H(2+\Delta_H)}} \\
ABC[G \bullet_{nv} H] &\leq \sqrt{2}[m_1 - \Delta_G] + \frac{\Delta_G}{\Delta_H}[m_2 - \Delta_H]\sqrt{2(\Delta_H-1)} \\
&+ 2\Delta_G\sqrt{\frac{\Delta_G+\Delta_H}{\Delta_G(2+\Delta_H)}} \\
&+ \Delta_G\Delta_H\sqrt{\frac{2}{2+\Delta_H}}.
\end{aligned}
$$

One can analogously compute the following

$$
\begin{aligned}
ABC[G \bullet_{nv} H] \geq \sqrt{2}[m_1 - \delta_G] + \frac{\delta_G}{\delta_H}[m_2 - \delta_H]\sqrt{2(\delta_H-1)} \\
+ 2\delta_G\sqrt{\frac{\delta_G+\delta_H}{\delta_G(2+\delta_H)}} + \delta_G\delta_H\sqrt{\frac{2}{2+\delta_H}}.
\end{aligned}
$$

□

Theorem 36. *Let G and H are two simple connected graphs, then the bounds for the SK_1 indeg index of $G \bullet_{nv} H$ are given by*

$$SK_1[G \bullet_{nv} H] \leq [m_1 - \Delta_G][\Delta_G + 2] + \Delta_G\Delta_H[m_2 - \Delta_H] + \Delta_G[\Delta_G + \Delta_H + 2] + \Delta_G\Delta_H[\Delta_H + 1].$$

$$SK_1[G \bullet_{nv} H] \geq [m_1 - \delta_G][\delta_G + 2] + \delta_G\delta_H[m_2 - \delta_H] + \delta_G[\delta_G + \delta_H + 2] + \delta_G\delta_H[\delta_H + 1].$$

Proof. Consider,

$$\begin{aligned} SK_1[G \bullet_{nv} H] &= \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in V[G], v \in I[G]}} \left[\frac{d_G(u) + 2}{2}\right] + \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u,v \in V[H]}} \left[\frac{d_H(u) + d_H(v)}{2}\right] \\ &+ \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in M[G \bullet_{nv} H], v \in V[G]}} \left[\frac{d_G(u) + (2 + d_H(v))}{2}\right] \\ &+ \sum_{\substack{uv \in E[G \bullet_{nv} H] \\ u \in M[G \bullet_{nv} H], v \in V[H]}} \left[\frac{(2 + d_H(u)) + d_H(v)}{2}\right] \\ &= 2[m_1 - d_G(S(u))]\left[\frac{d_G(u) + 2}{2}\right] \\ &+ d_G(S(u))[m_2 - d_H(S(u))]\left[\frac{d_H(u) + d_H(v)}{2}\right] \end{aligned}$$

$$+ 2d_G(S(u))\left[\frac{d_G(u) + d_H(v) + 2}{2}\right]$$

$$+ d_G(S(u))d_H(S(u))\left[\frac{2 + d_H(u) + d_H(v)}{2}\right]$$

$$\leq [m_1 - \Delta_G][\Delta_G + 2] + \Delta_G[m_2 - \Delta_H]\left[\frac{2\Delta_H}{2}\right]$$

$$+ \Delta_G[\Delta_G + \Delta_H + 2] + \Delta_G\Delta_H\left[\frac{2(\Delta_H + 1)}{2}\right]$$

$$SK_1[G \bullet_{nv} H] \leq [m_1 - \Delta_G][\Delta_G + 2] + \Delta_G\Delta_H[m_2 - \Delta_H]$$

$$+ \Delta_G[\Delta_G + \Delta_H + 2] + \Delta_G\Delta_H[\Delta_H + 1]$$

One can analogously compute the following

$$SK_1[G \bullet_{nv} H] \geq [m_1 - \delta_G][\delta_G + 2] + \delta_G\delta_H[m_2 - \delta_H]$$

$$+ \delta_G[\delta_G + \delta_H + 2] + \delta_G\delta_H[\delta_H + 1]$$

□

6.5 Subdivision-edge neighbourhood splice graph

Let $p_1 \in I(G)$ be the inserted vertex of $S(G)$ and let $y_1 \in V(H)$. Then the S-edge neighbourhood splice of G and H is denoted by $G \bullet_{ne} H$ that is obtained from $S(G)$ and two copies of H identifying the vertices

p_1. For $y_1 \in V(H)$, the union of the corresponding neighbourhood separated vertices p_1 of $S(G)$ (see below Figure 6.5).

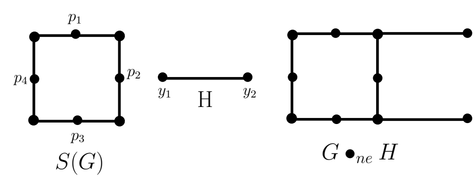

Figure 6.5: Subdivision-edge neighbourhood splice

Theorem 37. *Let G and H are two simple connected graphs, then the bounds for the inverse sum indeg index of $G \bullet_{ne} H$ are given by*

$$ISI[G \bullet_{ne} H] \leq \frac{4\Delta_G[m_1 - 2(\Delta_G - 1)]}{\Delta_G + 2} + \Delta_H[m_2 - \Delta_H] + \frac{8(\Delta_G + \Delta_H)(\Delta_G - 1)}{\Delta_G + \Delta_H + 2} + \frac{2\Delta_H^2(\Delta_G + \Delta_H)}{\Delta_G + 2\Delta_H}.$$

$$ISI[G \bullet_{ne} H] \geq \frac{4\delta_G[m_1 - 2(\delta_G - 1)]}{\delta_G + 2} + \delta_H[m_2 - \delta_H] + \frac{8(\delta_G + \delta_H)(\delta_G - 1)}{\delta_G + \delta_H + 2} + \frac{2\delta_H^2(\delta_G + \delta_H)}{\delta_G + 2\delta_H}.$$

Proof. Consider,

$$\begin{aligned}
ISI[G \bullet_{ne} H] &= \sum_{\substack{uv\in E[G\bullet_{ne}H]\\ u\in V[G],v\in I[G]}} \left[\frac{d_G(u).2}{d_G(u)+2}\right] + \sum_{\substack{uv\in E[G\bullet_{ne}H]\\ u,v\in V[H]}} \left[\frac{d_H(u).d_H(v)}{d_H(u)+d_H(v)}\right] \\
&+ \sum_{\substack{uv\in E[G\bullet_{ne}H]\\ u\in M[G\bullet_{ne}H],v\in I[G]}} \left[\frac{(d_G(u)+d_H(v)).2}{(d_G(u)+d_H(v))+2}\right] \\
&+ \sum_{\substack{uv\in E[G\bullet_{ne}H]\\ u\in M[G\bullet_{ne}H],v\in V[H]}} \left[\frac{(d_G(u)+d_H(w)).d_H(v)}{(d_G(u)+d_H(w))+d_H(v)}\right] \\
&= 2[m_1 - d_G(S(e))]\left[\frac{2d_G(u)}{d_G(u)+2}\right] \\
&+ 2[m_2 - d_H(S(u))]\left[\frac{d_H(u).d_H(v)}{d_H(u)+d_H(v)}\right] \\
&+ 2d_G(S(e))\left[\frac{2(d_G(u)+d_H(v))}{d_G(u)+d_H(v)+2}\right] \\
&+ 2d_H(S(u))\left[\frac{(d_G(u)+d_H(w)).d_H(v)}{d_G(u)+d_H(w)+d_H(v)}\right]. \\
ISI[G \bullet_{ne} H] &\leq \frac{4\Delta_G[m_1 - 2(\Delta_G - 1)]}{\Delta_G + 2} + \Delta_H[m_2 - \Delta_H] \\
&+ \frac{8(\Delta_G+\Delta_H)(\Delta_G - 1)}{\Delta_G+\Delta_H+2} + \frac{2\Delta_H^2(\Delta_G+\Delta_H)}{\Delta_G+2\Delta_H}.
\end{aligned}$$

One can analogously compute the following

$$ISI[G \bullet_{ne} H] \geq \frac{4\delta_G[m_1 - 2(\delta_G - 1)]}{\delta_G + 2} + \delta_H[m_2 - \delta_H]$$

$$+ \frac{8(\delta_G + \delta_H)(\delta_G - 1)}{\delta_G + \delta_H + 2} + \frac{2\delta_H^2(\delta_G + \delta_H)}{\delta_G + 2\delta_H}.$$

□

Theorem 38. *Let G and H are two simple connected graphs, then the bounds for the EM_1 index of $G \bullet_{ne} H$ are given by*

$$EM_1[G \bullet_{ne} H] \leq 2\Delta_G^2[m_1 - 2(\Delta_G - 1)] + 8[m_2 - \Delta_H][\Delta_H - 1]^2$$
$$+ 4[\Delta_G - 1][\Delta_G + \Delta_H]^2 + 2\Delta_H[\Delta_G + 2(\Delta_H - 1)]^2.$$
$$EM_1[G \bullet_{ne} H] \geq 2\delta_G^2[m_1 - 2(\delta_G - 1)] + 8[m_2 - \delta_H][\delta_H - 1]^2$$
$$+ 4[\delta_G - 1][\delta_G + \delta_H]^2 + 2\delta_H[\delta_G + 2(\delta_H - 1)]^2.$$

Proof. Consider,

$$EM_1[G \bullet_{ne} H] = \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in V[G], v \in I[G]}} [d_G(u) + 2 - 2]^2 + \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u,v \in V[H]}} [d_H(u) + d_H(v) - 2]^2$$
$$+ \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in M[G \bullet_{ne} H], v \in I[G]}} [(d_G(u) + d_H(v) + 2 - 2]^2$$
$$+ \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in M[G \bullet_{ne} H], v \in V[H]}} [(d_G(u) + d_H(w)) + d_H(v) - 2]^2.$$

$$
\begin{aligned}
&= 2[m_1 - d_G(S(e))][d_G(u)]^2 \\
&+ 2[m_2 - d_H(S(u))][d_H(u) + d_H(v) - 2]^2 \\
&+ 2d_G(S(e))[d_G(u) + d_H(v)]^2 \\
&+ 2d_H(S(u))[d_G(u) + d_H(w) + d_H(v) - 2]^2 \\
&\leq 2\Delta_G^2[m_1 - 1] + 2[m_2 - \Delta_H][\Delta_H + \Delta_H - 2]^2 \\
&+ 2[\Delta_G + \Delta_H]^2 + 2\Delta_H[\Delta_G + \Delta_H + \Delta_H - 2]^2 \\
&\leq 2\Delta_G^2[m_1 - 1] + 2[m_2 - \Delta_H][2\Delta_H - 2]^2 \\
&+ 2[\Delta_G + \Delta_H]^2 + 2\Delta_H[\Delta_G + 2\Delta_H - 2]^2 \\
EM_1[G \bullet_{ne} H] &\leq 2\Delta_G^2[m_1 - 2(\Delta_G - 1)] + 8[m_2 - \Delta_H][\Delta_H - 1]^2 \\
&+ 4[\Delta_G - 1][\Delta_G + \Delta_H]^2 + 2\Delta_H[\Delta_G + 2(\Delta_H - 1)]^2.
\end{aligned}
$$

One can analogously compute the following

$$
\begin{aligned}
EM_1[G \bullet_{ne} H] &\geq 2\delta_G^2[m_1 - 2(\delta_G - 1)] + 8[m_2 - \delta_H][\delta_H - 1]^2 \\
&+ 4[\delta_G - 1][\delta_G + \delta_H]^2 + 2\delta_H[\delta_G + 2(\delta_H - 1)]^2.
\end{aligned}
$$

□

Theorem 39. *Let G and H are two simple connected graphs, then the bounds for the atom-bond connectivite index of $G \bullet_{ne} H$ are given by*

$$ABC[G \bullet_{ne} H] \leq \sqrt{2}[m_1 - 2[\Delta_G - 1]] + 2\sqrt{2}[m_2 - \Delta_H]\left[\frac{\sqrt{\Delta_H - 1}}{\Delta_H}\right] + 2\sqrt{2}(\Delta_G - 1) + 2\sqrt{\Delta_H}\sqrt{\frac{\Delta_G + 2\Delta_H}{\Delta_G + \Delta_H}}.$$

$$ABC[G \bullet_{ne} H] \geq \sqrt{2}[m_1 - 2[\delta_G - 1]] + 2\sqrt{2}[m_2 - \delta_H]\left[\frac{\sqrt{\delta_H - 1}}{\delta_H}\right] + 2\sqrt{2}(\delta_G - 1) + 2\sqrt{\delta_H}\sqrt{\frac{\delta_G + 2\delta_H}{\delta_G + \delta_H}}.$$

Proof. Consider,

$$\begin{aligned} ABC[G \bullet_{ne} H] = & \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in V[G], v \in I[G]}} \left[\sqrt{\frac{d_G(u) + 2 - 2}{d_G(u).2}}\right] \\ & + \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u,v \in V[H]}} \left[\sqrt{\frac{d_H(u) + d_H(v) - 2}{d_H(u).d_H(v)}}\right] \\ & + \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in M[G \bullet_{ne} H], v \in I[G]}} \left[\sqrt{\frac{(d_G(u) + d_H(v)) + 2 - 2}{(d_G(u) + d_H(v)).2}}\right] \\ & + \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in M[G \bullet_{ne} H], v \in V[H]}} \left[\sqrt{\frac{(d_G(u) + d_H(w)) + d_H(v) - 2}{(d_G(u) + d_H(w)).d_H(v)}}\right] \\ & = 2[m_1 - d_G(S(e))]\left[\sqrt{\frac{d_G(u)}{2d_G(u)}}\right] \end{aligned}$$

$$
\begin{aligned}
&+ 2[m_2 - d_H(S(u))]\left[\sqrt{\frac{d_H(u) + d_H(v) - 2}{d_H(u).d_H(v)}}\right] \\
&+ 2d_G(S(e))\left[\sqrt{\frac{(d_G(u) + d_H(v))}{2(d_G(u) + d_H(v))}}\right] \\
&+ 2d_H(S(u))\left[\sqrt{\frac{d_G(u) + d_H(w) + d_H(v) - 2}{d_H(v)(d_G(u) + d_H(w))}}\right]. \\
&= \frac{2}{\sqrt{2}}[m_1 - d_G(S(e))] + 2[m_2 - d_H(S(u))]\left[\sqrt{\frac{d_H(u) + d_H(v) - 2}{d_H(u).d_H(v)}}\right] \\
&\leq \sqrt{2}[m_1 - 1] + 2[m_2 - \Delta_H]\sqrt{\frac{\Delta_H + \Delta_H - 2}{\Delta_H.\Delta_H}} + \sqrt{2} \\
&+ 2\Delta_H\sqrt{\frac{\Delta_G + \Delta_H + \Delta_H - 2}{\Delta_H.(\Delta_G + \Delta_H)}} \\
&\leq \sqrt{2}[m_1 - 1] + 2[m_2 - \Delta_H]\sqrt{\frac{2\Delta_H - 2}{\Delta_H^2}} + \sqrt{2} \\
&+ 2\Delta_H\sqrt{\frac{\Delta_G + 2\Delta_H - 2}{\Delta_H.(\Delta_G + \Delta_H)}}
\end{aligned}
$$

$$
\begin{aligned}
ABC[G \bullet_{ne} H] \leq{}& \sqrt{2}[m_1 - 2[\Delta_G - 1]] + 2\sqrt{2}[m_2 - \Delta_H]\left[\frac{\sqrt{\Delta_H - 1}}{\Delta_H}\right] \\
&+ 2\sqrt{2}(\Delta_G - 1) + 2\sqrt{\Delta_H}\sqrt{\frac{\Delta_G + 2\Delta_H}{\Delta_G + \Delta_H}}.
\end{aligned}
$$

One can analogously compute the following

$$
\begin{aligned}
ABC[G \bullet_{ne} H] \geq{}& \sqrt{2}[m_1 - 2[\delta_G - 1]] + 2\sqrt{2}[m_2 - \delta_H]\left[\frac{\sqrt{\delta_H - 1}}{\delta_H}\right] \\
&+ 2\sqrt{2}(\delta_G - 1) + 2\sqrt{\delta_H}\sqrt{\frac{\delta_G + 2\delta_H}{\delta_G + \delta_H}}.
\end{aligned}
$$

□

Theorem 40. *Let G and H are two simple connected graphs, then the bounds for the SK index of $G \bullet_{ne} H$ are given by*

$$SK_1[G \bullet_{ne} H] \leq [m_1 - 1][2 + \Delta_G] + 2\Delta_H[m_2 - \Delta_H] + [\Delta_G + \Delta_H + 2] + \Delta_H[\Delta_G + 2\Delta_H]$$

and

$$SK_1[G \bullet_{ne} H] \geq [m_1 - 1][\delta_G + 2] + 2\delta_H[m_2 - \delta_H] + [\delta_G + \delta_H + 2] + \delta_H[\delta_G + 2\delta_H]$$

Proof. Consider,

$$\begin{aligned} SK_1[G \bullet_{ne} H] &= \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in V[G], v \in I[G]}} \left[\frac{d_G(u) + 2}{2}\right] + \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u, v \in V[H]}} \left[\frac{d_H(u) + d_H(v)}{2}\right] \\ &+ \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in M[G \bullet_{ne} H], v \in I[G]}} \left[\frac{(d_G(u) + d_H(v)) + 2}{2}\right] \\ &+ \sum_{\substack{uv \in E[G \bullet_{ne} H] \\ u \in M[G \bullet_{ne} H], v \in V[H]}} \left[\frac{(d_G(u) + d_H(w)) + d_H(v)}{2}\right] \\ &= 2[m_1 - d_G(S(e))] \left[\frac{d_G(u) + 2}{2}\right] \\ &+ 2[m_2 - d_H(S(u))] \left[\frac{d_H(u) + d_H(v)}{2}\right] \\ &+ 2d_G(S(e)) \left[\frac{d_G(u) + d_H(v) + 2}{2}\right] \end{aligned}$$

$$+ 2d_H(S(u))\left[\frac{d_G(u) + d_H(w) + d_H(v)}{2}\right]$$

$$\leq [m_1 - 1][\Delta_G + 2] + [m_2 - \Delta_H][\Delta_H + \Delta_H]$$

$$+ [\Delta_G + \Delta_H + 2] + \Delta_H[\Delta_G + \Delta_H + \Delta_H]$$

$$SK_1[G \bullet_{ne} H] \leq [m_1 - 1][2 + \Delta_G] + 2\Delta_H[m_2 - \Delta_H] + [\Delta_G + \Delta_H + 2]$$

$$+ \Delta_H[\Delta_G + 2\Delta_H].$$

One can analogously compute the following

$$SK_1[G \bullet_{ne} H] \geq [m_1 - 1][\delta_G + 2] + 2\delta_H[m_2 - \delta_H] + [\delta_G + \delta_H + 2] + \delta_H[\delta_G + 2\delta_H].$$

$\square$

Chapter 7

S- Corona operations of standard graphs in terms of degree sequences

7.1 Introduction and preliminaries

The degree sequences (DS)s of G is $DS(G) = \{\lambda_1^{\xi_1}, \lambda_2^{\xi_2}, \lambda_3^{\xi_3}, ..., \lambda_n^{\xi_n}\}$ can be obtained by degree of v_i of G in ascending or descending order [48]. In 1981, Bollobas [8] started the study DSs and Tyshkevich et.al., established a correspondence between DSs of graph and some structural properties of this graph in same year [47].

Definition 7.1.1. ***S*- vertex corona:** Consider two graphs G *and* H with vertex sets p_1 and p_2 and edge sets q_1 and q_2 respectively. The S- vertex corona of graphs G and H with disjoint vertex sets $V(G)$ and $V(H)$ and edge sets $E(G)$ and $E(H)$ is obtained one $S(G)$ and $|V(G)|$ number of copies H, by joining i^{th} vertex in $V(G)$ to each vertex of i^{th} copy H [[9], [20]]. Then, $|V(G \odot_S H)| = p_1(1+p_2)+q_1$ and $|E(G \odot_S H)| = 2q_1 + p_1(q_2+p_2)$.

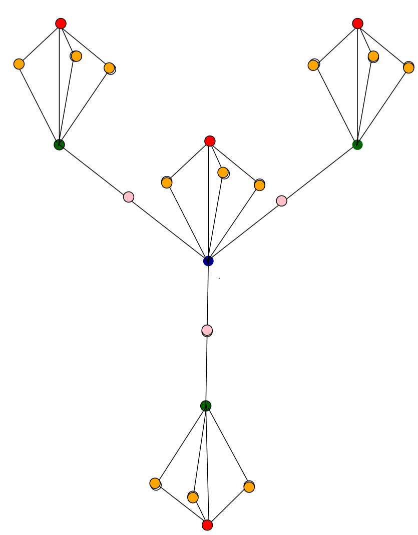

Figure 7.1: Subdivision-vertex corona of S_4 and S_4

Definition 7.1.2. The S- edge corona of graphs G and H with disjoint vertex sets $V(G)$ and $V(H)$ and edge sets $E(G)$ and $E(H)$ is obtained from $S(G)$ (Subdivision graph of G) and $|E(G)|$ copies of H, by joining the i^{th} vertex of $I(G)$ ($I(G)$ is the inserted vertices in $S(G)$) to each vertex in the i^{th} copy of H. Then, $|V(G \ominus_S H)| = p_1 + q_1(1 + p_2)$ and $|E(G \ominus_S H)| = q_1(2 + q_2 + p_2)$.

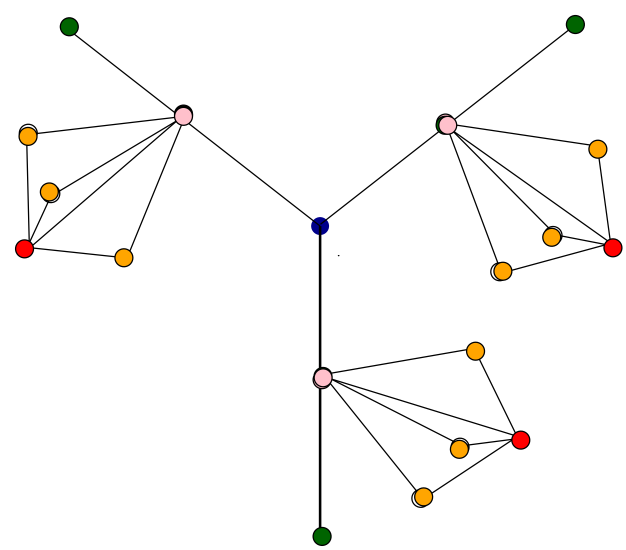

Figure 7.2: Subdivision-edge corona of S_4 and S_4

Definition 7.1.3. The S**- edge neighbourhood corona** of graphs G and H with disjoint vertex sets $V(G)$ and $V(H)$ and edge sets $E(G)$ and $E(H)$ is obtained from $S(G)$ and $|E(G)|$ number of copies H, by joining neighbours of i^{th} vertex in $I(G)$ ($I(G)$ is the inserted vertices in G) to each vertex of i^{th} copy H. Then, $|V(G \ominus_{nS} H)| = p_1 + q_1(1 + p_2)$ and $|E(G \ominus_{nS} H)| = q_1(2 + q_2 + 2p_2)$.

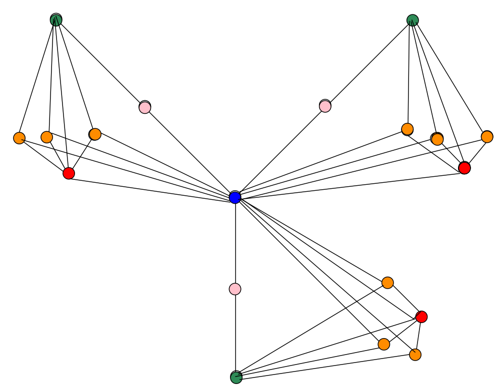

Figure 7.3: Subdivision-edge neighbourhood corona of S_4 and S_4

7.2 Main Results

In this section, the DSs of the S- vertex(edge) corona and S- edge neighbourhood corona of graphs G_1 and G_2 chosen from P_n, K_n, C_n, S_n, $K_{n,m}$ and r-regular graphs are obtained.

Theorem 41. *The DSs of all possible S-vertex corona of the path, complete, cycle, star, complete bipartite and r-regular graphs.*

Proof. First, the proof of $S_n \odot_S S_m$ is observing. Let $DS(S_n) = \{1^{n-1}, (n-1)^1\}$ and $DS(S_m) = \{1^{m-1}, (m-1)^1\}$. To understand situation see Figure 7.1.

There are two types of vertices in each of S_n and S_m. Therefore there are $2 + 2 + 1 = 5$ types of vertices in $S_n \odot_S S_m$. The first type is the centre vertex (blue) of S_n which are connected with the $(n-1)$ vertices (pink) in $I(S_n)$ and mn-vertices in n-copies of S_m. Each of these $(n-1)$ vertices add $(1+m)$ to the DSs of $S_n \odot_S S_m$. Therefore they add $(1+m)^{n-1}$.

The second type of vertices (green) of S_n which are connected with the vertex (pink) of $I(S_n)$ and nm-vertices in n-copies of S_m. Each of these one vertex add $(n+m-1)$ to the DSs of $S_n \odot_S S_m$. Therefore they add $(n+m-1)^1$.

The third type of centre vertices (red) of n^{th}-copies of S_m which are connected with the $(m-1)$ end vertices (orange) in n^{th} copy of S_m and n^{th} vertex in S_n. Each of these $(m-1)n$ vertices add 2 to the DSs of $S_n \odot_S S_m$. Therefore they add $2^{(m-1)n}$.

The fourth type of end vertices (orange) of n^{th}-copy of S_m which are connected with the centre vertex (red) of n^{th}-copy of S_m and n^{th} vertex in S_n. Each of these n vertices add m to the DSs of $S_n \odot_S S_m$. Therefore they add m^n.

The fifth type of vertices (pink) in $I(S_n)$ (Inserted vertices) which are connected with the centre vertex (blue) and $(n-1)$ end vertices (green) in S_n. Each of these $(n-1)$ vertices add 2 to the DSs of $S_n \odot_S S_m$. Therefore they add 2^{n-1}. Thus,

$$DS(S_n \odot_S S_m) = \{(1+m)^{n-1}, (n+m-1)^1, 2^{(m-1)n}, m^n, 2^{n-1}\}.$$

Table 7.1: Degree Sequences of S-vertex corona for path, Complete, Cycle, Star, Complete Bipartite and r-regular graphs.

G	H	$DS(G \odot_S H)$
P_n	P_m	$\{(1+m)^2, (2+m)^{n-2}, 2^{n-1}, 2^{2n}, 3^{n(m-2)}\}$
P_n	K_m	$\{(1+m)^2, (2+m)^{n-2}, 2^{n-1}, m^{mn}\}$
P_n	C_m	$\{(1+m)^2, (2+m)^{n-2}, 2^{n-1}, 3^{mn}\}$
P_n	S_m	$\{(1+m)^2, (2+m)^{n-2}, 2^{n-1}, 2^{n(m-1)}, m^n\}$
P_n	$K_{m,o}$	$\{(1+m+o)^2, (2+m+o)^{n-2}, 2^{n-1}, (m+1)^{no},$ (o+1)$^{nm}\}$
P_n	r-regular with m-vertices	$\{(1+m)^2, (2+m)^{n-2}, 2^{n-1}, (r+1)^{nm}\}$
K_n	P_m	$\{(n+m-1)^n, 2^{n(n-1)/2}, 2^{2n}, 3^{n(m-2)}\}$
K_n	K_m	$\{(n+m-1)^n, 2^{n(n-1)/2}, m^{nm}\}$
K_n	C_m	$\{(n+m-1)^n, 2^{n(n-1)/2}, 3^{nm}\}$
K_n	S_m	$\{(n+m-1)^n, 2^{n(n-1)/2}, 2^{n(m-1)}, m^n\}$
K_n	$K_{m,o}$	$\{(n+m+o-1)^n, 2^{n(n-1)/2}, (m+1)^{no}, (o+1)^{nm}\}$
K_n	r-regular with m-vertices	$\{(n+m-1)^n, 2^{n(n-1)/2}, (r+1)^{nm}\}$
C_n	P_m	$\{(2+m)^n, 2^n, 2^{2n}, 3^{n(m-2)}\}$
C_n	K_m	$\{(2+m)^n, 2^n, m^{nm}\}$
C_n	C_m	$\{(2+m)^n, 2^n, 3^{nm}\}$

C_n	S_m	$\left\{(2+m)^n, 2^n, 2^{n(m-1)}, m^n\right\}$
C_n	$K_{m,o}$	$\{(2+m+o)^n, 2^n, (m+1)^{no}, (o+1)^{nm}\}$
C_n	r-regular with m-vertices	$\{(2+m)^n, 2^n, (r+1)^{nm}\}$
S_n	P_m	$\left\{(1+m)^{n-1}, 2^{n-1}, (n+m-1), 3^{n(m-2)}, 2^{2n}\right\}$
S_n	K_m	$\left\{(1+m)^{n-1}, (n+m-1), 2^{n-1}, m^{mn}\right\}$
S_n	C_m	$\left\{(1+m)^{n-1}, (n+m-1), 2^{n-1}, 3^{nm}\right\}$
S_n	S_m	$\left\{(1+m)^{n-1}, (n+m-1), 2^{n-1}, 2^{n(m-1)}, m^n\right\}$
S_n	$K_{m,o}$	$\left\{(1+m+o)^{n-1}, (n+m+o-1), 2^{n-1},\right.$ (m+1)^{on}, $\left.(o+1)^{mn}\right\}$
S_n	r-regular with m-vertices	$\left\{(1+m)^{n-1}, (n+m-1), 2^{n-1}, (r+1)^{nm}\right\}$
$K_{m,n}$	P_o	$\left\{(n+o)^m, 2^{mn}, 2^{2(m+n)}, (m+o)^n, 3^{(o-2)(n+m)}\right\}$
$K_{m,n}$	K_o	$\left\{(m+o)^n, (n+o)^m, 2^{mn}, o^{o(m+n)}\right\}$
$K_{m,n}$	C_o	$\left\{(m+o)^n, (n+o)^m, 2^{mn}, 3^{o(m+n)}\right\}$
$K_{m,n}$	S_o	$\left\{(m+o)^n, (n+o)^m, 2^{mn}, 2^{(o-1)(m+n)}, o^{(m+n)}\right\}$
$K_{m,n}$	$K_{r,s}$	$\{(m+r+s)^n, (n+r+s)^m, 2^{mn},$ (r+1)$^{s(m+n)}$, $(s+1)^{r(m+n)}\}$

$K_{m,n}$	r-regular with o-vertices	$\{(m+o)^n, (n+o)^m, 2^{mn}, (r+1)^{o(m+n)}\}$
r-regular with n-vertices	P_m	$\{(r+m)^n, 2^{nr/2}, 2^{2n}, 3^{(m-2)}\}$
r-regular with n-vertices	K_m	$\{(r+m)^n, 2^{nr/2}, m^{mn}\}$
r-regular with n-vertices	C_m	$\{(r+m)^n, 2^{nr/2}, 3^{mn}\}$
r-regular with n-vertices	S_m	$\{(r+m)^n, 2^{nr/2}, 2^{n(m-1)}, m^n\}$
r-regular with n-vertices	$K_{m,o}$	$\{(r+m+o)^n, 2^{nr/2}, (m+1)^{no}, (o+1)^{nm}\}$
r_1-regular with n-vertices	r_2-regular with m-vertices	$\{(r_1+m)^n, 2^{nr_1/2}, (r_2+1)^{mn}\}$

$\square$

Theorem 42. *The DSs of all possible S-edge corona of the path, complete, cycle, star, complete bipartite and r-regular graphs.*

Proof. First, the proof of $S_n \ominus_S S_m$ is observing. Let $DS(S_n) = \{1^{n-1}, (n-1)^1\}$ and $DS(S_m) = \{1^{m-1}, (m-1)^1\}$. To understand situation see Figure 7.2.

There are two types of vertices in each of S_n and S_m. Therefore there are $2+2+1 = 5$ types of vertices in $S_n \ominus_S S_m$. The first type is the centre vertex (blue) of S_n which are connected with the $(n-1)$ vertices (pink) in $I(S_n)$. Each of these $(n-1)$ vertices add 1 to the DSs of $S_n \ominus_S S_m$. Therefore they add 1^{n-1}.

The second type of vertices (green) of S_n which are connected with the vertex (pink) of $I(S_n)$. Each of these one vertex add $(n-1)$ to the DSs of $S_n \ominus_S S_m$. Therefore they add $(n-1)^1$.

The third type of centre vertices (red) of $(n-1)^{th}$-copies of S_m which are connected with the $(m-1)$ end vertices (orange) in $(n-1)^{th}$ copy of S_m and vertex in $I(S_n)$. Each of these $(m-1)(n-1)$ vertices add 2 to the DSs of $S_n \ominus_S S_m$. Therefore they add $2^{(m-1)(n-1)}$.

The fourth type of end vertices (orange) of $(n-1)^{th}$-copy of S_m which are connected with the centre vertex (red) in $(n-1)^{th}$-copy of S_m and $(n-1)^{th}$ vertex in $I(S_n)$. Each of these $(n-1)$ vertices add m to the DSs of $S_n \ominus_S S_m$. Therefore they add m^{n-1}.

The fifth type of vertices (pink) in $I(S_n)$ (Inserted vertices) which are connected with the centre vertex (blue) and $(n-1)$ end vertices (green) in S_n and each vertex of $(n-1)^{th}$ copy of S_m. Each of these $(n-1)$ vertices add $(2+m)$ to the DSs of $S_n \ominus_S S_m$. Therefore they add $(2+m)^{n-1}$. Thus,

$$DS(S_n \ominus_S S_m) = \{1^{n-1}, (n-1), 2^{(m-1)(n-1)}, m^{n-1}, (2+m)^{n-1}\}.$$

Table 7.2: Degree Sequences of S-edge corona for path, Complete, Cycle, Star, Complete Bipartite and r-regular graphs.

G	H	$DS(G \ominus_S H)$
P_n	P_m	$\{1^2, 2^{n-2}, (2+m)^{n-1}, 2^{2(n-1)}, 3^{(n-1)(m-2)}\}$
P_n	K_m	$\{1^2, (2+m)^{n-1}, 2^{n-2}, m^{m(n-1)}\}$
P_n	C_m	$\{1^2, 2^{n-2}, (2+m)^{n-1}, 3^{m(n-1)}\}$
P_n	S_m	$\{1^2, 2^{n-2}, (2+m)^{n-1}, 2^{(n-1)(m-1)}, m^{n-1}\}$
P_n	$K_{m,o}$	$\{1^2, 2^{n-2}, (2+m+o)^{n-1}, (m+1)^{o(n-1)}, (o+1)^{m(n-1)}\}$
P_n	r-regular with m-vertices	$\{1^2, (r+1)^{m(n-1)}, 2^{n-2}, (2+m)^{n-1}\}$
K_n	P_m	$\{(n-1)^n, (2+m)^{n(n-1)/2}, 2^{n(n-1)}, 3^{n(n-1)(m-2)/2}\}$
K_n	K_m	$\{(n-1)^n, (2+m)^{n(n-1)/2}, m^{mn(n-1)/2}\}$
K_n	C_m	$\{(n-1)^n, (2+m)^{n(n-1)/2}, 3^{mn(n-1)/2}\}$
K_n	S_m	$\{(n-1)^n, (2+m)^{n(n-1)/2}, 2^{n(n-1)(m-1)/2}, m^{n(n-1)/2}\}$
K_n	$K_{m,o}$	$\{(\text{n-1})^n, (2+m+o)^{n(n-1)/2},$ $(\text{m+1})^{on(n-1)/2}, (o+1)^{mn(n-1)/2}\}$
K_n	r-regular with m-vertices	$\{(n-1)^n, (2+m)^{n(n-1)/2}, (r+1)^{mn(n-1)/2}\}$
C_n	P_m	$\{2^n, 3^{n(m-2)}, (2+m)^n, 2^{2n}\}$
C_n	K_m	$\{2^n, m^{mn}, (2+m)^n\}$

C_n	C_m	$\{2^n, (2+m)^n, 3^{mn}\}$
C_n	S_m	$\left\{2^n, (2+m)^n, 2^{n(m-1)}, m^n\right\}$
C_n	$K_{m,o}$	$\{2^n, (2+m+o)^n, (m+1)^{no}, (o+1)^{mn}\}$
C_n	r-regular with m-vertices	$\{2^n, (2+m)^n, (r+1)^{mn}\}$
S_n	P_m	$\left\{1^{n-1}, (n-1), (2+m)^{n-1}, 2^{2(n-1)}, 3^{(n-1)(m-2)}\right\}$
S_n	K_m	$\left\{1^{n-1}, m^{m(n-1)}, (n-1), (2+m)^{n-1}\right\}$
S_n	C_m	$\left\{1^{n-1}, 3^{m(n-1)}, (n-1), (2+m)^{n-1}\right\}$
S_n	S_m	$\left\{1^{n-1}, (2+m)^{n-1}, (n-1), 2^{(m-1)(n-1)}, m^{n-1}\right\}$
S_n	$K_{m,o}$	$\{1^{n-1}, (n-1), (2+m+o)^{n-1},$ (m+1)$^{o(n-1)}, (o+1)^{m(n-1)}\}$
S_n	r-regular with m-vertices	$\left\{1^{n-1}, (n-1), (2+m)^{n-1}, (r+1)^{m(n-1)}\right\}$

$K_{m,n}$	P_o	$\left\{n^m, m^n, (2+o)^{mn}, 2^{2mn}, 3^{mn(o-2)}\right\}$
$K_{m,n}$	K_o	$\{n^m, m^n, (2+o)^{mn}, o^{omn}\}$
$K_{m,n}$	C_o	$\{n^m, m^n, (2+o)^{mn}, 3^{omn}\}$
$K_{m,n}$	S_o	$\left\{n^m, m^n, (2+o)^{mn}, 2^{(o-1)mn}, o^{mn}\right\}$
$K_{m,n}$	$K_{r,s}$	$\{n^m, m^n, (2+r+s)^{mn}, (r+1)^{mns}, (s+1)^{mnr}\}$
$K_{m,n}$	r-regular with o-vertices	$\{n^m, m^n, (2+o)^{mn}, (r+1)^{mno}\}$
r-regular with n-vertices	P_m	$\left\{r^n, (2+m)^{nr/2}, 2^{nr}, 3^{nr(m-2)/2}\right\}$
r-regular with n-vertices	K_m	$\left\{r^n, (2+m)^{nr/2}, m^{mnr/2}\right\}$
r-regular with n-vertices	C_m	$\left\{r^n, (2+m)^{nr/2}, 3^{mnr/2}\right\}$
r-regular with n-vertices	S_m	$\left\{r^n, (2+m)^{nr/2}, 2^{nr(m-1)/2}, m^{nr/2}\right\}$
r-regular with n-vertices	$K_{m,o}$	$\left\{r^n, (2+m+o)^{nr/2}, (m+1)^{onr/2}, (o+1)^{nmr/2}\right\}$
r_1-regular with n-vertices	r_2-regular with m-vertices	$\left\{r_1^n, (2+m)^{nr_1/2}, (r_2+1)^{mnr_1/2}\right\}$

□

Theorem 43. *The DSs of all possible S-edge neighbourhood corona of the path, complete, cycle, star, complete bipartite and r-regular graphs.*

Proof. First, the proof of $S_n \ominus_{nS} S_m$ is observing. Let $DS(S_n) = \{1^{n-1}, (n-1)^1\}$ and $DS(S_m) = \{1^{m-1}, (m-1)^1\}$. To understand situation see Figure 7.3.

There are two types of vertices in each of S_n and S_m. Therefore there are $2+2+1=5$ types of vertices in $S_n \ominus_{nS} S_m$. The first type is the centre vertex (blue) of S_n which are connected with the $(n-1)$ vertices (pink) in $I(S_n)$ and each vertex in $(n-1)$ copies of S_n. Each of these one vertices add $(n-1+(n-1)m)$ to the DSs of $S_n \ominus_{nS} S_m$. Therefore they add $(n-1+(n-1)m)$.

The second type of vertices (green) of S_n which are connected with the vertex (pink) of $I(S_n)$ and each vertex in one copy of S_n. Each of these $(n-1)$ vertices add $(1+m)$ to the DSs of $S_n \ominus_{nS} S_m$. Therefore they add $(1+m)^{n-1}$.

The third type of centre vertices (red) of $(n-1)^{th}$-copies of S_m which are connected with the $(m-1)$ end vertices (orange) in corresponding $(n-1)^{th}$ copies of S_m and vertex (blue and green) in S_n which are neighbourhood of $(n-1)$ vertex of $I(S_n)$. Each of these $(n-1)$ vertices add $(m+1)$ to the DSs of $S_n \ominus_{nS} S_m$. Therefore they add $(m+1)^{(n-1)}$.

The fourth type of end vertices (orange) of $(n-1)^{th}$-copies of S_m which are connected with the centre vertex (red) in corresponding $(n-1)^{th}$-copies of S_m and vertices (blue and green) of S_n which are neighbourhood of $(n-1)^{th}$ vertex of $I(S_n)$. Each of these $(m-1)(n-1)$ vertices add 3 to the DSs of $S_n \ominus_{nS} S_m$. Therefore they add $3^{(m-1)(n-1)}$.

The fifth type of vertices (pink) in $I(S_n)$ (Inserted vertices) which are connected with the centre vertex (blue) and one end vertices (green) in S_n. Each of these $(n-1)$ vertices add 2 to the DSs of $S_n \ominus_{nS} S_m$.

Therefore they add 2^{n-1}. Thus,

$$DS(S_n \ominus_{nS} S_m) = \{(n-1+(n-1)m), (1+m)^{n-1}, (m+1)^{(n-1)}, 3^{(m-1)(n-1)}, 2^{n-1}\}.$$

Table 7.3: Degree Sequences of S-edge neighbourhood corona for path, Complete, Cycle, Star, Complete Bipartite and r-regular graphs.

G	H	$DS(G \ominus_{nS} H)$
P_n	P_m	$\{(1+m)^2, (2+2m)^{n-2}, 2^{n-1}, 3^{2(n-1)}, 4^{(n-1)(m-2)}\}$
P_n	K_m	$\{(1+m)^2, (2+2m)^{n-2}, 2^{n-1}, (m+1)^{m(n-1)}\}$
P_n	C_m	$\{(1+m)^2, (2+2m)^{n-2}, 2^{n-1}, 4^{m(n-1)}\}$
P_n	S_m	$\{(1+\text{m})^2, (2+2m)^{n-2}, 2^{n-1}, 3^{(n-1)(m-1)}, (m+1)^{n-1}\}$
P_n	$K_{m,o}$	$\{(1+(\text{m+o}))^2, (2+2(m+o))^{n-2}, 2^{n-1}, (\text{m+2})^{on}, (o+2)^{mo}\}$
P_n	r-regular with m-vertices	$\{(1+m)^2, (2+2m)^{n-2}, 2^{n-1}, (r+2)^{(n-1)m}\}$
K_n	P_m	$\{(n-1+(n-1)m)^n, 2^{n(n-1)/2}, 3^{n(n-1)}, 4^{n(n-1)(m-2)/2}\}$
K_n	K_m	$\{(n-1+(n-1)m)^n, 2^{n(n-1)/2}, (m+1)^{mn(n-1)/2}\}$
K_n	C_m	$\{(n-1+(n-1)m)^n, 2^{n(n-1)/2}, 4^{mn(n-1)/2}\}$
K_n	S_m	$\{(\text{n-1+(n-1)m})^n, 2^{n(n-1)/2}, 3^{n(n-1)(m-1)/2}, (m+1)^{n(n-1)/2}\}$

K_n	$K_{m,o}$	$\{$(n-1+(n-1)m)$^n, 2^{n(n-1)/2}$, (m+2)$^{on(n-1)/2}, (o+2)^{mn(n-1)/2}\}$
K_n	r-regular with m-vertices	$\{(n-1+(n-1)m)^n, 2^{n(n-1)/2}, (r+2)^{mn(n-1)/2}\}$
C_n	P_m	$\{(2+2m)^n, 2^n, 3^{2n}, 4^{n(m-2)}\}$
C_n	K_m	$\{(2+2m)^n, 2^n, (m+1)^{mn}\}$
C_n	C_m	$\{(2+2m)^n, 2^n, 4^{mn}\}$
C_n	S_m	$\{(2+2m)^n, 2^n, 3^{n(m-1)}, (m+1)^n\}$
C_n	$K_{m,o}$	$\{(2+2(m+o))^n, 2^n, (m+2)^{no}, (o+2)^{mn}\}$
C_n	r-regular with m-vertices	$\{(2+2m)^n, 2^n, (r+2)^{mn}\}$
S_n	P_m	$\{$(1+m)$^{n-1}, 2^{n-1}, (n-1+(n-1)m)$, $4^{(n-1)(m-2)}, 3^{2(n-1)}\}$
S_n	K_m	$\{$(n-1+(n-1)m),(1+m)$^{n-1}$, $2^{n-1}, (m+1)^{m(n-1)}\}$
S_n	C_m	$\{$(n-1+(n-1)m),(1+m)$^{n-1}$, $4^{m(n-1)}, 2^{n-1}\}$

S_n	S_m	$\{(1+\text{m})^{n-1}, 2^{n-1}, (n-1+(n-1)m),$ $(\text{m}+1)^{n-1}, 3^{(m-1)(n-1)}\}$
S_n	$K_{m,o}$	$\{(1+\text{m}+\text{o})^{n-1}, (n-1+(m+o)(n-1)), 2^{n-1},$ $(\text{m}+2)^{o(n-1)}, (o+2)^{m(n-1)}\}$
S_n	r-regular with m-vertices	$\{(\text{m}+1)^{n-1}, (n-1+m(n-1)),$ $2^{n-1}, (r+2)^{m(n-1)}\}$
$K_{m,n}$	P_o	$\{(n(o+1))^m, (m(o+1))^n, 2^{mn}, 3^{2mn}, 4^{mn(o-2)}\}$
$K_{m,n}$	K_o	$\{(n(o+1))^m, (m(o+1))^n, 2^{mn}, ((o+1))^{omn}\}$
$K_{m,n}$	C_o	$\{(n(o+1))^m, (m(o+1))^n, 2^{mn}, 4^{omn}\}$
$K_{m,n}$	S_o	$\{(\text{n}(\text{o}+1))^m, (m(o+1))^n, 2^{mn},$ $3^{(o-1)mn}, (o+1)^{mno}\}$
$K_{m,n}$	$K_{r,s}$	$\{(\text{n}+(\text{r}+\text{s})\text{n})^m, (m+(r+s)m)^n, 2^{nm},$ $(\text{r}+2)^{mns}, (s+2)^{mnr}\}$
$K_{m,n}$	r-regular with o-vertices	$\{(n(o+1))^m, ((o+1)m)^n, 2^{nm}, (r+2)^{mno}\}$

r-regular with n-vertices	P_m	$\{(r+rm)^n, 2^{nr/2}, 3^{nr}, 4^{nr(m-2)/2}\}$
r-regular with n-vertices	K_m	$\{(r+rm)^n, 2^{nr/2}, (m+1)^{mnr/2}\}$
r-regular with n-vertices	C_m	$\{(r+rm)^n, 2^{nr/2}, 4^{mnr/2}\}$
r-regular with n-vertices	S_m	$\{(r+rm)^n, 2^{nr/2}, 3^{nr(m-1)/2}, (m+1)^{nr/2}\}$
r-regular with n-vertices	$K_{m,o}$	$\{(r+r(m+o))^n, 2^{nr/2}, (m+2)^{onr/2}, (o+2)^{nmr/2}\}$
r_1-regular with n-vertices	r_2-regular with m-vertices	$\{(r_1+r_1m)^n, 2^{nr_1/2}, (r_2+2)^{mnr_1/2}\}$

$\square$

Chapter 8

The Degree sequences of S-corona graphs

8.1 Preliminaries

In this chapter, we obtain the DS of the S- vertex corona, S- edge corona, S- vertex neighbourhood corona and S- edge neighbourhood corona of any given number of simple connected graphs. First we start with graphs G H, obtain the $DS(G \odot_S H)$, $DS(G \ominus_S H)$ and $DS(G \ominus_{nS} H)$ and using mathematical induction, we obtain the general formula for $G_1 \odot_S G_2 \odot_S ... \odot_S G_k$, $G_1 \ominus_S G_2 \ominus_S ... \ominus_S G_k$, $G_1 \odot_{nS} G_2 \odot_{nS} ... \odot_{nS} G_k$ and $G_1 \ominus_{nS} G_2 \ominus_{nS} ... \ominus_{nS} G_k$ in terms of the number of vertices of G_is.

8.2 Generalization for the DSs of the S-vertex corona

Theorem 44. *Let G and H be two simple connected graphs with DSs*

$$DS(G) = \{\lambda_{11}^{\xi_{11}}, \lambda_{12}^{\xi_{12}}, ..., \lambda_{1k_1}^{\xi_{1k_1}}\}$$

and

$$DS(H)=\{\lambda_{21}^{\xi_{21}}, \lambda_{22}^{\xi_{22}}, ..., \lambda_{2k_2}^{\xi_{2k_2}}\}$$

respectively.

Proof. The DS of the S- vertex corona of the two graphs G and H is

$$DS(G \odot_S H) = \{(\lambda_{11} + r_2)^{\xi_{11}}, (\lambda_{12} + r_2)^{\xi_{12}}, ..., (\lambda_{1k_1} + r_2)^{\xi_{1k_1}}, 2^{s_1},$$
$$(\lambda_{21} + 1)^{r_1\xi_{21}}, (\lambda_{22} + 1)^{r_1\xi_{22}}, ..., (\lambda_{2k_2} + 1)^{r_1\xi_{2k_2}}\}.$$

Note that to obtain $DS(G \odot_S H)$, we add r_2 to each λ_{1x} where $1 \leq x \leq k_1$, without changing the powers ξ_{1x} , add number 1 to each λ_{2x}, where $1 \leq x \leq k_2$, with changing the powers as $r_1\xi_{2x}$ and 2^{s_1}.

Let us consider $DS(P_l) = \{1^2, 2^{l-2}\}$ and $DS(P_m) = \{1^2, 2^{m-2}\}$, we will find the DS of $P_l \odot_S P_m$. Let r_1 and r_2 be the vertices of P_l and P_m respectively.

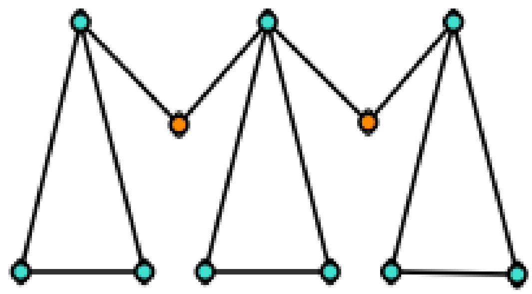

Figure 8.1: Subdivision-vertex corona of P_3 and P_2

As $\lambda_{11} = 1, \xi_{11} = 2, \lambda_{12} = 2, \xi_{12} = 1, \lambda_{21} = 1, \xi_{21} = 2$ by the definition of S- vertex corona.

We have,

$$\begin{aligned} DS(P_3 \odot_S P_2) &= \{1^2, 2^1\} \odot_S \{1^2\} \\ &= \{(1+2)^2, (2+2)^1, 2^2, (1+1)^{3\times 2}\} \\ &= \{2^8, 3^2, 4^1\}. \end{aligned}$$

8.3 Generalization for the DSs of the S-edge corona

Theorem 45. *Let G and H be two simple connected graphs with DSs*

$$DS(G) = \{\lambda_{11}^{\xi_{11}}, \lambda_{12}^{\xi_{12}}, ..., \lambda_{1k_1}^{\xi_{1k_1}}\}$$

and

$$DS(H)=\{\lambda_{21}^{\xi_{21}}, \lambda_{22}^{\xi_{22}}, ..., \lambda_{2k_2}^{\xi_{2k_2}}\}$$

respectively.

Proof. The DS of the S- edge corona of the two graphs G and H is

$$DS(G \ominus_S H) = \{\lambda_{11}^{\xi_{11}}, \lambda_{12}^{\xi_{12}}, ..., \lambda_{1k_1}^{\xi_{1k_1}}, (2 + r_2)^{s_1}, (\lambda_{21} + 1)^{s_1\xi_{21}},$$
$$(\lambda_{22} + 1)^{s_1\xi_{22}}, ..., (\lambda_{2k_2} + 1)^{s_1\xi_{2k_2}}\}.$$

Note that to obtain $DS(G \ominus_S H)$, we write $DS(G)$ without changing, add number 1 to each λ_{2x} where $1 \leq x \leq k_2$, with changing the powers as $s_1\xi_{2x}$ and $(2 + r_2)^{s_1}$. □

Let us consider $DS(P_l) = \{1^2, 2^{l-2}\}$ and $DS(P_m) = \{1^2, 2^{m-2}\}$, we will find the DS of $P_l \ominus_S P_m$. Let r_1 and r_2 be the vertices of P_l and P_m respectively.

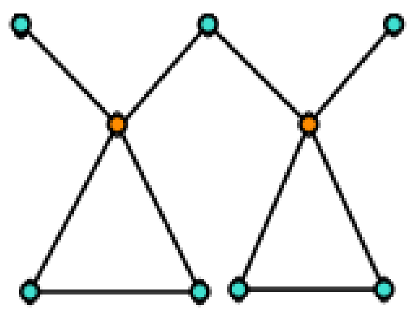

Figure 8.2: Subdivision-edge corona of P_3 and P_2

As $\lambda_{11} = 1$, $\xi_{11} = 2$, $\lambda_{12} = 2$, $\xi_{12} = 1$, $\lambda_{21} = 1$, $\xi_{21} = 2$ by the definition of S- edge corona.

We have,

$$\begin{aligned} DS(P_3 \ominus_S P_2) &= \{1^2, 2^1\} \ominus_S \{1^2\} \\ &= \{1^2, 2^1, (2+2)^2, (1+1)^{2\times 2}\} \\ &= \{1^2, 2^5, 4^2\}. \end{aligned}$$

8.4 Generalization for the DSs of the S-vertex neighbourhood corona

Theorem 46. *Let G and H be two simple connected graphs with DSs*

$$DS(G) = \{\lambda_{11}^{\xi_{11}}, \lambda_{12}^{\xi_{12}}, ..., \lambda_{1k_1}^{\xi_{1k_1}}\}$$

and

$$DS(H)=\{\lambda_{21}^{\xi_{21}}, \lambda_{22}^{\xi_{22}}, ..., \lambda_{2k_2}^{\xi_{2k_2}}\}$$

respectively.

Proof. The DS of the S- vertex neighbourhood corona of the two graphs G and H is

$$DS(G \odot_{nS} H) = \{\lambda_{11}^{\xi_{11}}, \lambda_{12}^{\xi_{12}}, ..., \lambda_{1k_1}^{\xi_{1k_1}}, (2 + 2r_2)^{s_1},$$

$$(\lambda_{21} + \lambda_{11})^{\xi_{21}\xi_{11}}, (\lambda_{22} + \lambda_{11})^{\xi_{22}\xi_{11}}, ..., (\lambda_{2k_2} + \lambda_{11})^{\xi_{2k_2}\xi_{11}},$$

$$(\lambda_{21}+\lambda_{12})^{\xi_{21}\xi_{12}}, (\lambda_{22}+\lambda_{12})^{\xi_{22}\xi_{12}}, ..., (\lambda_{2k_2}+\lambda_{12})^{\xi_{2k_2}\xi_{12}},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$(\lambda_{21}+\lambda_{1k_1})^{\xi_{21}\xi_{1k_1}}, (\lambda_{22}+\lambda_{1k_1})^{\xi_{22}\xi_{1k_1}}, ..., (\lambda_{2k_2}+\lambda_{1k_1})^{\xi_{1k_1}\xi_{2k_2}}\}.$$

Note that to obtain $DS(G \odot_{nS} H)$, we write $DS(G)$ without changing, add each λ_{1x} to λ_{2y} where $1 \leq x \leq k_1$ and $1 \leq y \leq k_2$, with changing the powers $r_1\xi_{2y}$ and $(2+2r_2)^{s_1}$. □

Let us consider $DS(P_l) = \{1^2, 2^{l-2}\}$ and $DS(P_m) = \{1^2, 2^{m-2}\}$, we will find the DS of $P_l \odot_{nS} P_m$. Let r_1 and r_2 be the vertices of P_l and P_m respectively.

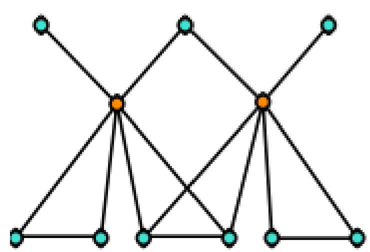

Figure 8.3: Subdivision-vertex neighbourhood corona of P_3 and P_2

As $\lambda_{11} = 1$, $\xi_{11} = 2$, $\lambda_{12} = 2$, $\xi_{12} = 1$, $\lambda_{21} = 1$, $\xi_{21} = 2$ by the definition of S- vertex neighbourhood corona.

We have,

$$\begin{aligned} DS(P_3 \odot_{nS} P_2) &= \{1^2, 2^1\} \odot_{nS} \{1^2\} \\ &= \{1^2, 2^1, (2+4)^2, (1+1)^{2\times 2}, (1+2)^{2\times 1}\} \\ &= \{1^2, 2^5, 3^2, 6^2\}. \end{aligned}$$

8.5 Generalization for the DSs of the S-edge neighbourhood corona

Theorem 47. *Let G and H be two simple connected graphs with DSs*

$$DS(G) = \{\lambda_{11}^{\xi_{11}}, \lambda_{12}^{\xi_{12}}, ..., \lambda_{1k_1}^{\xi_{1k_1}}\}$$

and

$$DS(H)=\{\lambda_{21}^{\xi_{21}}, \lambda_{22}^{\xi_{22}}, ..., \lambda_{2k_2}^{\xi_{2k_2}}\}$$

respectively.

Proof. The DS of the S- edge neighbourhood corona of the two graphs

G and H is

$$DS(G \ominus_{nS} H) = \{(\lambda_{11} + \lambda_{11}r_2)^{\xi_{11}}, (\lambda_{12} + \lambda_{12}r_2)^{\xi_{12}}, ..., (\lambda_{1k_1} + \lambda_{1k_1}r_2)^{\xi_{1k_1}}, 2^{s_1},$$
$$(\lambda_{21} + 2)^{s_1\xi_{21}}, (\lambda_{22} + 2)^{s_1\xi_{22}}, ..., (\lambda_{2k_2} + 2)^{s_1\xi_{2k_2}}\}.$$

Note that to obtain $DS(G \ominus_{nS} H)$, we add $\lambda_{1x}r_2$ to each λ_{1x} where $1 \leq x \leq k_1$, without changing the powers ξ_{1x} , add number 2 to each λ_{2x} where $1 \leq x \leq k_2$, with changing the powers as $s_1\xi_{2x}$ and 2^{s_1}.

□

Let us consider $DS(P_l) = \{1^2, 2^{l-2}\}$ and $DS(P_m) = \{1^2, 2^{m-2}\}$, we will find the DS of $P_l \ominus_{nS} P_m$. Let r_1 and r_2 be the vertices of P_l and P_m respectively.

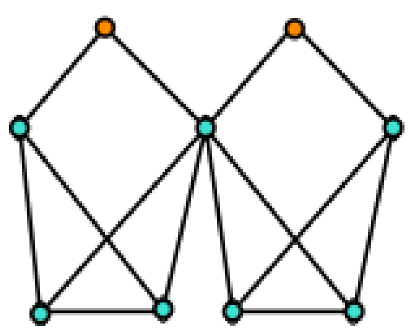

Figure 8.4: Subdivision-edge neighbourhood corona of P_3 and P_2

As $\lambda_{11} = 1$, $\xi_{11} = 2$, $\lambda_{12} = 2$, $\xi_{12} = 1$, $\lambda_{21} = 1$, $\xi_{21} = 2$ by the definition of S- edge neighbourhood corona.

We have,

$$\begin{aligned} DS(P_3 \ominus_{nS} P_2) &= \{1^2, 2^1\} \ominus_{nS} \{1^2\} \\ &= \{(1+1(2))^2, (2+2(2))^1, 2^2, (1+2)^{2\times 2}\} \\ &= \{2^2, 3^6, 6^1\}. \end{aligned}$$

Now we take the S- vertex corona, S- edge corona, S- vertex neighbourhood corona and S- edge neighbourhood corona of l simple connected graphs $G_1, G_2, G_3, ..., G_l$, where $l \geq 2$ is a finite integer. The DS of $G_1 \odot_S G_2 \odot_S ... \odot_S G_l$, $G_1 \ominus_S G_2 \ominus_S ... \ominus_S G_l$, $G_1 \odot_{nS} G_2 \odot_{nS} ... \odot_{nS} G_l$ and $G_1 \ominus_{nS} G_2 \ominus_{nS} ... \ominus_{nS} G_l$ is given as follows.

Theorem 48. *Let $G_1, G_2, G_3, ..., G_l$ be l simple connected graphs. Let G_i have n_i vertices for $i = 1, 2, ..., l$. Also let the DS of G_i be*

$$DS(G) = \{\lambda_{i1}^{\xi_{i1}}, \lambda_{i2}^{\xi_{i2}}, ..., \lambda_{ik_1}^{\xi_{ik_1}}\}$$

Proof. The DS of the S-vertex corona of $G_1, G_2, G_3, ..., G_l$ is

$$\begin{aligned}
DS(G_1 \odot_S G_2 \odot_S ... \odot_S G_l) = \{&(\lambda_{11} + r_2 + r_3 + ... + r_l)^{\xi_{11}}, ...,\\
&(\lambda_{1k_1} + r_2 + r_3 + ... + r_l)^{\xi_{1k_1}},\\
&(\lambda_{21} + 1 + r_3 + r_4 + ... + r_l)^{r_1\xi_{21}}, ...,\\
&(\lambda_{2k_2} + 1 + r_3 + r_4 + ... + r_l)^{r_1\xi_{2k_2}},\\
&(\lambda_{31} + 1 + r_4 + r_5 + ... + r_l)^{|V(G_1 \odot_S G_2)|\xi_{31}}, ...,\\
&(\lambda_{3k_3} + 1 + r_4 + r_5 + ... + r_l)^{|V(G_1 \odot_S G_2)|\xi_{3k_3}},\\
&...,\\
&(\lambda_{(l-1)1} + 1 + r_l)^{|V(G_1 \odot_S G_2 \odot_S ... \odot_S G_{(l-2)})|\xi_{(l-1)1}}, ...,\\
&(\lambda_{(l-1)k_{(l-1)}} + 1 + r_l)^{|V(G_1 \odot_S G_2 \odot_S ... \odot_S G_{(l-2)})|\xi_{(l-1)k_{(l-1)}}},\\
&(\lambda_{l1} + 1)^{|V(G_1 \odot_S G_2 \odot_S ... \odot_S G_{(l-1)})|\xi_{l1}}, ...,\\
&(\lambda_{lk_l} + 1)^{|V(G_1 \odot_S G_2 \odot_S ... \odot_S G_{(l-1)})|\xi_{lk_l}},\\
&(2 + r_3 + r_4 + ... + r_l)^{s_1},
\end{aligned}$$

$$(2 + r_4 + r_5 + ... + r_l)^{|E(G_1 \odot_S G_2)|},$$

$$..., (2)^{|E(G_1 \odot_S G_2 \odot_S ... \odot_S G_{(l-1)})|}\}.$$

□

Theorem 49. *Let $G_1, G_2, G_3, ..., G_l$ be l simple connected graphs. Let G_i have n_i vertices for $i = 1, 2, ..., l$. Also let the DS of G_i be*

$$DS(G_i) = \{\lambda_{i1}^{\xi_{i1}}, \lambda_{i2}^{\xi_{i2}}, ..., \lambda_{ik_i}^{\xi_{ik_i}}\}.$$

Proof. The DS of the S-edge corona of $G_1, G_2, G_3, ..., G_l$ is

$$DS(G_1 \ominus_S G_2 \ominus_S ... \ominus_S G_l) = \{\lambda_{11}^{\xi_{11}}, \lambda_{12}^{\xi_{12}}, ..., \lambda_{1k_1}^{\xi_{1k_1}}, (\lambda_{21} + 1)^{s_1\xi_{21}}, ..., (\lambda_{2k_2} + 1)^{s_1\xi_{2k_2}}$$

$$(\lambda_{31} + 1)^{|E(G_1 \ominus_S G_2)|\xi_{31}}, ..., (\lambda_{3k_3} + 1)^{|E(G_1 \ominus_S G_2)|\xi_{3k_3}}$$

$$...,$$

$$(\lambda_{(l-1)1}+1)^{|E(G_1\ominus_S G_2\ominus_S \ldots \ominus_S G_{(l-2)})|\xi_{(l-1)1}}, \ldots,$$

$$(\lambda_{(l-1)k_{(l-1)}}+1)^{|E(G_1\ominus_S G_2\ominus_S \ldots \ominus_S G_{(l-2)})|\xi_{(l-1)k_{(l-1)}}},$$

$$(\lambda_{l1}+1)^{|E(G_1\ominus_S G_2\ominus_S \ldots \ominus_S G_{(l-1)})|\xi_{l1}}, \ldots,$$

$$(\lambda_{lk_l+1})^{|E(G_1\ominus_S G_2\ominus_S \ldots \ominus_S G_{(l-1)})|\xi_{lk_l}},$$

$$(2+r_2)^{s_1}, (2+r_3)^{|E(G_1\ominus_S G_2)|}, \ldots,$$

$$(2+r_l)^{|E(G_1\ominus_S G_2\ominus_S \ldots \ominus_S G_{(l-1)})|}\}.$$

□

Theorem 50. *Let $G_1, G_2, G_3, \ldots, G_l$ be l simple connected graphs. Let G_i have n_i vertices for $i = 1, 2, \ldots, l$. Also let the DS of G_i be*

$$DS(G_i) = \{\lambda_{i1}^{\xi_{i1}}, \lambda_{i2}^{\xi_{i2}}, \ldots, \lambda_{ik_i}^{\xi_{ik_i}}\}.$$

Proof. The DS of the S-vertex neighbourhood corona of $G_1, G_2, G_3, \ldots, G_l$ is

$$DS(G_1 \odot_{nS} G_2 \odot_{nS} ... \odot_{nS} G_l) = \{\lambda_{11}^{\xi_{11}}, \lambda_{12}^{\xi_{12}}, ..., \lambda_{1k_1}^{\xi_{1k_1}}, (\lambda_{21}+\lambda_{11})^{\xi_{21}\xi_{11}}, ...,$$

$$(\lambda_{2k_2}+\lambda_{11})^{\xi_{2k_2}\xi_{11}}$$

$$...,$$

$$(\lambda_{21}+\lambda_{1k_1})^{\xi_{21}\xi_{1k_1}}, ..., (\lambda_{2k_2}+\lambda_{1k_1})^{\xi_{2k_2}\xi_{1k_1}}$$

$$...,$$

$$(\lambda_{l1}+\lambda_{11})^{\xi_{l1}\xi_{11}}, ..., (\lambda_{lk_l}+\lambda_{11})^{\xi_{lk_l}\xi_{11}},$$

$$...,$$

$$(\lambda_{l1}+\lambda_{1k_1}+\lambda_{2k_2}+...+\lambda_{(l-1)k_{(l-1)}})^{\xi_{l1}\xi_{1k_1}\xi_{2k_2}\cdots\xi_{(l-1)k_{(l-1)}}},$$

$$..., (\lambda_{lk_l}+\lambda_{1k_1}+\lambda_{2k_2}+...$$

$$+\lambda_{(l-1)k_{(l-1)}})^{\xi_{lk_l}\xi_{1k_1}\xi_{2k_2}\cdots\xi_{(l-1)k_{(l-1)}}},$$

$$(2+2r_2)^{s_1}, (2+2r_3)^{|E(G_1\odot_{nS}G_2)|}, ...,$$

$$(2+2r_l)^{|E(G_1\odot_{nS}G_2\odot_{nS}\cdots\odot_{nS}G_{(l-1)})|}\}.$$

□

Theorem 51. *Let $G_1, G_2, G_3, ..., G_l$ be l simple connected graphs. Let G_i have n_i vertices for $i = 1, 2, ..., l$. Also let the DS of G_i be*

$$DS(G_i)=\{\lambda_{i1}^{\xi_{i1}}, \lambda_{i2}^{\xi_{i2}}, ..., \lambda_{ik_i}^{\xi_{ik_i}}\}.$$

Proof. The DS of the S-edge neighbourhood corona of $G_1, G_2, G_3, ..., G_l$ is

$$\begin{aligned} DS(G_1 \ominus_{nS} G_2 \ominus_{nS} ... \ominus_{nS} G_l) = \{&(..((\lambda_{11} + r_2\lambda_{11}) + (\lambda_{11} + r_2\lambda_{11})r_3)r_4 + ..)^{\xi_{11}} \\ &, ..., (..((\lambda_{1k_1} + r_2\lambda_{1k_1}) + (\lambda_{1k_1} + r_2\lambda_{1k_1})r_3)r_4 + ..)^{\xi_{1k_1}} \\ &.., \\ &(\lambda_{l1} + 2)^{|E(G_1 \ominus_{nS} G_2 \ominus_{nS} ... \ominus_{nS} G_{(l-1)})|\xi_{l1}}, ..., \\ &(\lambda_{lk_l} + 2)^{|E(G_1 \ominus_{nS} G_2 \ominus_{nS} ... \ominus_{nS} G_{(l-1)})|\xi_{lk_l}}, \\ &((2 + 2r_2) + (2 + 2r_2)r_3) \\ &+ ((2 + 2r_2) + (2 + 2r_2)r_3)r_4 \end{aligned}$$

$$+\ ...)^{|E(G_1 \ominus_{nS} G_2 \ominus_{nS} ... \ominus_{nS} G_{(l-1)})|}\}.$$

$\square$

Chapter 9

Conclusions and Scope for Future Work

- The first Chapter is of introductory nature. The first part of the Chapter-1 is loyal to a study of the basic terminology and notations in the graph theory.

- In chapter 2, the Gourava index of four operation on graphs in terms of first and second Zagreb index are obtained.

- In chapter 3, we computed adriatic indices for subdivision, line and derived graph Dutch windmill graph.

- In chapter 4, established the general expression for some adriatic indices and Sanskruti index of carbon nanocones $[CNC_m^n]$. It is clear that, these results have benefits to forecast physical properties of elemental chemical compounds and useful for determining the Physio-chemical properties of alkanes.

- In chapter 5, certain degree based adriatic indices of graph operators of triglyceride are computed without using computers.

- In chapter 6, we have study the lower and upper bounds for the topological indices in terms of the graph size and maximum or minimum degree of splice graph are obtained.

- In chapter 7, we determine the S- vertex(edge) and S-edge neighbourhood corona of standard graphs(Path, Complete, Cycle, Star, Complete bipartite and r-regular graphs).

- In chapter 8, we study the S-vertex corona, S-edge corona, S-vertex neighbourhood corona and S-edge neighbourhood corona of number of simple graphs. From these results we get information about graph that are useful to understand the problems corresponding to the graphs.

9.1 Future Work

For confines work, we pose the following problems for continuation work which were interesting:

- Analysing the general graphs for different graph operators.
- The upper and lower bounds on topological indices.
- The general formula for graph operators of degree sequences on simple connected graphs .
- The molecular structures such as carbon graphite, crystal cubic carbon structures and benzin ring respectively with most useful indices.

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