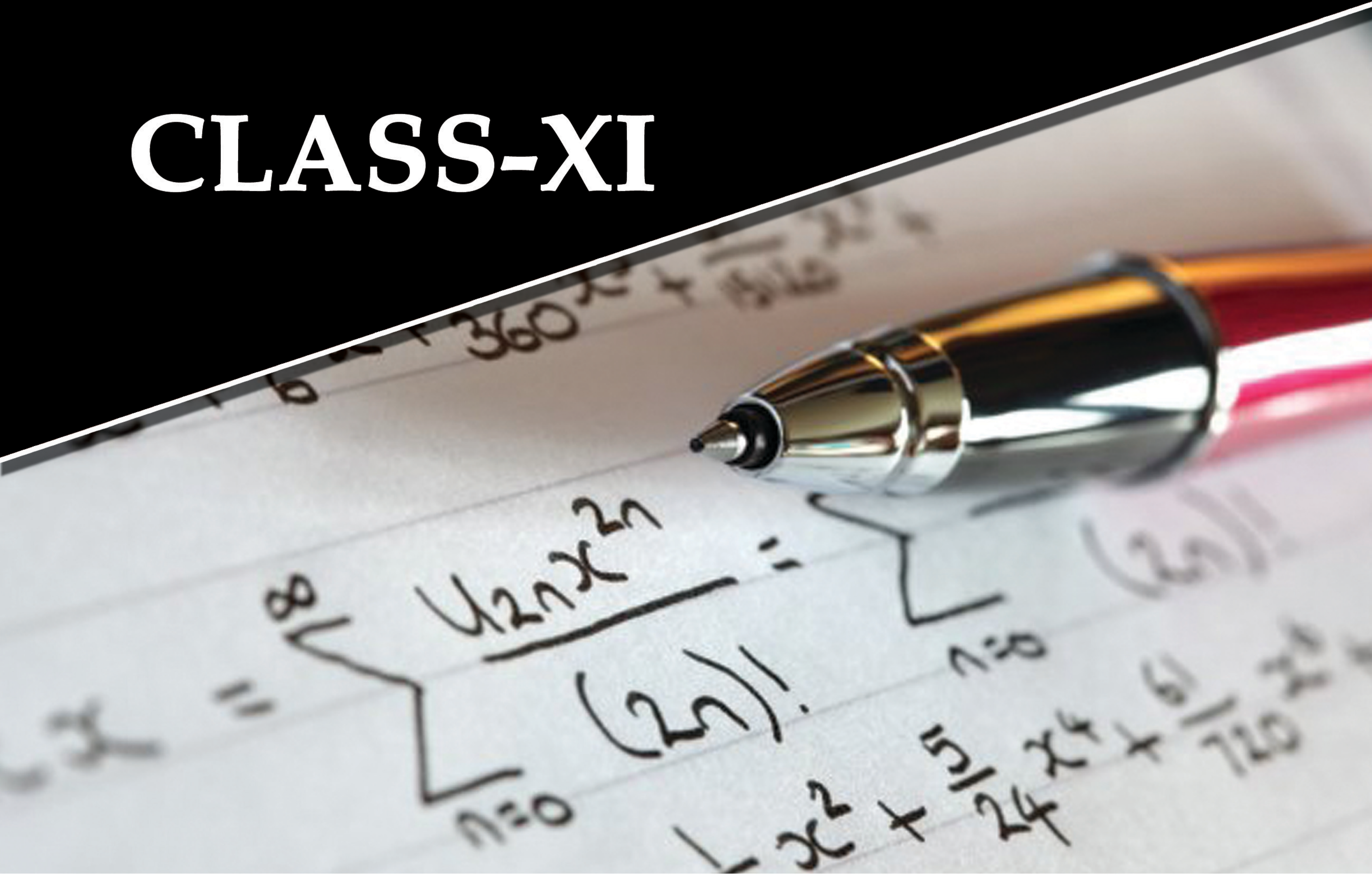
CLASS-XI



Work Book Cum Question Bank with Answers

MATHEMATICS

SCHEDULED CASTES & SCHEDULED TRIBES
RESEARCH & TRAINING INSTITUTE (SCSTRTI)
ST & SC DEVELOPMENT DEPARTMENT
BHUBANESWAR

WORK BOOK CUM
QUESTION BANK WITH ANSWERS

MATHEMATICS

CLASS - XI

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Contents

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MATHEMATICS (1st Year) Syllabus

UNIT - I : Sets and Functions

1. Sets

Sets and their representations. Empty set, Finite and Infinite sets, Equal sets, Subsets, Subsets of a set of real numbers especially intervals (with notations), Power set, Universal set, Venn diagrams, Union and Intersection of sets, Difference of sets, Complement of a set, Properties of Complement of Sets, Practical Problems based on sets.

2. Relations & Functions

Ordered pairs, Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the sets of real (upto R × R). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special kind of relation from one set to another. Pictorial representation of a function, domain co-domain and range of a function. Real valued functions, domain and range of these functions: Constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer function, with their graphs. Sum, difference, product and quotients of functions.

3. Trigonometric Functions

Positive and negative angles. Measuring angles in radians and in degrees and conversion of one into other. Definition of trigonometric functions with the help of unit circle. Truth of $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing sin (x ± y) and cos (X± y) in terms of sinx, siny, cosx & cosy and their simple application. Deducing identities like the following :

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y},\ \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2},\ \cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2},$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2},\ \cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2},$$

Identities related to sin 2x, cos 2x, tan 2x, sin 3x, cos 3x and tan 3x. Trigonometric equations Principal solution, General solution of trigonometric equations of the type sin x = sin y, cos x = cos y and tan x = tan y. Proof and Simple applications of sine and cosine formula.

UNIT - II : Algebra

1. Principle of Mathematical Induction

Process of the proof by induction, motivation the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.

2. Complex Numbers and Quadratic Equations

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quardratic equations; Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex system. Square root of a complex number, cube roots of unity and its properties.

3. Linear Inequalities

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical solution of system of linear inequalities in two variables.

4. Permutations and Combinations

Fundamental principle of counting, factorial n. (n!), Permutations and combinations, derivation of formulae and their connections, simple applications.

5. Binomial Theorem

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications.

6. Sequence and Series

Sequence and Series, Arithmetic Progression (A.P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P, sum of n terms of a G.P., Arithmetic and Geometric series, infinite G.P. and its sum, geometric mean (G.M.), Harmonic (mean) relation between A.M., GM. and H.M., Formula for the following special sum :

Arithmetico-Geometric Series, Exponential Series, Logarithmic Series, Binomial Series.

UNIT-III : Co-ordinate Geometry

1. Straight Lines

Brief recall of two dimensional geometry from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line : parallel to axis, point-slope form, slope-intercept form, two-point form, intercept form and normal form. General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line, Shifting of Origin.

2. Conic Sections

Sections of a cone : circles, ellipse, parabola, hyperbola; a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section; Standard equations and simple properties of Circle, parabola, ellipse and hyperbola.

3. Introduction to Three-dimensional Geometry

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

UNIT-IV : Calculus

1. Limits and Derivatives

Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions, trigonometric, exponential and logarithmic functions. Definition of derivative, relate it to slope of tangent of a curve, derivative of sum, difference, product and quotient of functions. The derivative of polynomial and trigonometric functions.

UNIT-V : Mathematical Reasoning

1. Mathematical Reasoning

Mathematically acceptable statements. Connecting words/phrases-consolidating the understanding of "if and only if (necessary and sufficient) condition," "implies", "and/ or", "implied by", "and", "or", "there exists" and their use through variety of examples related to real life and Mathematics. Validating the statements involving the connecting words, difference between contradiction, converse and contrapositive.

UNIT-VI : Statistics and Probability

1. Statistics

Measures of dispersion; Range, mean deviation, variance and standard deviation of ungrouped/ grouped data. Analysis of frequency distributions with equal means but different variances.

2. Probability

Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and'and 'or'events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with the theories of earlier classes. Probability of an event. Probability of 'not', 'and' 'or' events.

QUESTION PATTERN OF CHSE

Unit	Topic	Marks	No. of Periods
I	Sets and Functions	29	60
II	Algebra	37	70
III	Co-ordinate Geometry	13	40
IV	Calculus	06	30
V	Mathematical Reasoning	03	10
VI	Statistics and Probability	12	30
	Total	**100**	**220**

GENERAL IMSTRUCTIONS :

1. All questions are compulsory in Group A, which are very short answer type questions. All questions in the group are to be answered in one word, one sentences or as per exact requirement of the question. (1 x 10 = 10 Marks)

2. Group-B contain 5 (five) questions and each question have 5 bits, out of which only 3 bits are to be answered (Each bit caries 4 Marks) (4 x15 = 60 Marks)

3. Group-C contains 5 (five) questions and each question contains 2/3 bits, out of which only 1 (one) bit is to be answered. Each bit caries 6 (six) Mark (6 x 5 =30 Marks)

CHAPTERS IN BRIEF

SETS

1. Set is a collection of well defined objects. The objects are called elements.

   Sets are denoted by Capital Letters A,B,C ... and elements are denoted by small letters a,b,c, ...

2. Sets are represented in two forms.

   (i) Roster form or Tabular Form

   (ii) Set Builder Form

   Set of all positive even number

   = {2, 4, 6, 8, ...} (Roster Form)

   = {x : x is an even integer}

   (set builder form).

3. **Empty Set :** A set which does not contains any element is called an empty set.

   It is denoted by $\varnothing$

   $\because \ \varnothing = \{\ \ \} = \{x : x \neq x\}$

4. **Cardinality of a Set :** The number of elements of a set is called the cardinality of a set. The cardinality of a Set A is written as $|A|$ or 0 (A).

5. **Equivalent Sets (Similar Sets) :** Two sets are said to be equivalent if they have the same cardinality.

   Two Sets A and B are equivalent if $|A| = |B|$.

6. **Subset :** It every element of the Set A is also an element of the Set B, then A is said to be subset of B. It is denoted by $A \subset B$.

   Note : If $x \in A \Rightarrow x \in B$, then $A \subset B$.

   Proper Subset : If every element of the Set A is also an element of the Set B and in B, there exists at least one element which is not an element of A, then A is said to be a proper subset of B. It is deonoted by $A \underset{\neq}{\subset} B$.

7. **Equal Sets :** Two Sets A and B are said to be equal if every element of A is an element of the Set B and every element of the Set B is also an element of the Set A. It is written as A = B.

   Thus $A = B$ if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.

   i.e. $A = B$ if $x \in A \Leftrightarrow x \in B$.

   Note : If $A \subset B$ and $B \subset A$, then $A = B$.

8. **Universal Set :** It all the Sets under consideration are subsets of a particular set, then this particular set is called the universal set. It is denoted by E.

9. **Comparable Sets :** Two non empty sets A and B are said to be comparable if either $A \subset B$ or $B \subset A$ or $A = B$.

   Example - 1 : $A = \{a, b, c\}$, $B = \{a, b, c, d\}$ are comparable sets.

   Example - 2 : A = {1,2,3}, B={2,3,4} are not comparable as neither $A \subset B$ nor $B \subset A$ nor $A \neq B$.

10. **Power Set :** The Set of all subsets of a Set is called the power set of the set. If A be a set, then its power set is written as P(A).

    Note : If $|A| = n$ then $|P(A)| = 2^n$.

11. **Union of Sets :** The union of two Sets A and B is a set whose elements belong to either A or B or both. It is denoted by $A \bigcup B$.

    $\therefore \ A \bigcup B = \{x : x \in A \text{ or } x \in B\}$.

    Example :

    If A = {2, 3, 4}, B = {4, 5, 6}

    then $A \bigcup B = \{x : x \in A \text{ or } x \in B\}$

    = {2,3,4,5,6}

Notes :

(i) $A \bigcup A = A$

(ii) $A \bigcup \varnothing = A$

(iii) $A \bigcup E = E$

(iv) $A \subset A \bigcup B$ and $B \subset A \bigcup B$

(v) $x \in A \bigcup B \Leftrightarrow x \in A$ or $x \in B$

(vi) If $A_1, A_2, ..., A_n$ be n number of sets, then their union is written as $A_1 \bigcup A_2 \bigcup ... \bigcup A_n$.

It is denoted by $\bigcup_{i=1}^{n} A_i$

$\therefore \bigcup_{i=1}^{n} A_i = A_1 \bigcup A_2 \bigcup ... \bigcup A_n$.

12. **Intersectin of Sets :** If A and B be two sets then their intersection which is denoted by $A \bigcap B$ is a set whose elements are in both A and B.

$\because A \bigcap B = \{x \in A \text{ and } x \in B\}$.

Example -

If A = {2, 3, 4}, B = {4, 5, 6}

then $A \bigcap B = \{x : x \in A \text{ and } x \in B\} = \{4\}$.

Notes :

(i) $A \bigcap B = A$

(ii) $A \bigcap \varnothing = \varnothing$

(iii) $A \bigcap E = A$

(iv) $A \bigcap B \subset A$ and $A \bigcap B \subset B$

(v) $x \in A \bigcap B \Leftrightarrow x \in A$ and $x \in B$

(vi) If $A_1, A_2, ...,$ An be n number of sets, then their interesection is $A_1 \bigcap A_2 ... \bigcap A_n$.

It is denoted by $\bigcap_{i=1}^{n} A_i$.

$\therefore \bigcap_{i=1}^{n} A_i = A_1 \bigcap A_2 \bigcap ... \bigcap A_n$.

13. **Disjoint Sets :** Two Sets A and B are said to be disjoint if they have no common elements i.e. $A \bigcap B = \varnothing$.

14. **Difference of two sets :** If A and B be two sets then their difference which is denoted by A - B is a Set whose elements are in A but does not belong to B.

$\therefore A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly $B - A = \{x : x \in B \text{ and } x \notin A\}$

15. **Complement of a Set :** If A be a Set and E be the universal set, then the complement of A is a a set whose elements are in E but not in A. It is denoted by A^c or A^1.

$\therefore A^c$ or $A^1 = E - A$

$= \{x : x \in E \text{ and } x \notin A\}$.

16. **Symmetric difference of two sets :** If A and B be two sets then the set which is the union of A - B and B - A is called the symmetric difference of two sets. It is denoted by $A \Delta B$.

$\therefore A \Delta B = (A - B) \bigcup (B - A)$

$= \{x \in A \text{ or } x \in B\} \text{ and } x \notin A \bigcap B\}$.

17. **Algebra of Sets :**

(i) **Commutative Law :**

If A and B are two sets, then

(a) $A \bigcup B = B \bigcup A$

(b) $A \bigcap B = B \bigcap A$.

(ii) **Associative Law :**

If A, B and C be two sets, then

(a) $(A \bigcup B) \bigcup C = A \bigcup (B \bigcup C)$

(b) $(A \bigcap B) \bigcap C = A \bigcap (B \bigcap C)$

(iii) **Distributive Law :**

If A, B and C be three sets, then

(a) $A \bigcup (B \bigcap C) = (A \bigcup B) \bigcap (A \bigcup C)$

(b) $A \bigcap (B \bigcup C) = (A \bigcap B) \bigcup (A \bigcap C)$

(iv) **De Morgan's Law :**

If A and B are two sets, then

(a) $(A \bigcup B)' = A' \bigcap B'$

(b) $(A \bigcap B)' = A' \bigcup B'$

18. **Product of two sets :** If A and B are two sets then their product which is denoted by A x B is a set of order pairs (x, y) such that $x \in A$ and $y \in B$.

$\therefore A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$

Example : If A = {2, 3, 4}, B = {4, 5, 6}

then A x B = {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}.

Note :

Order Pair : A pair of numbers maintaining an order in called an order pair. An order pair containing a and b is written as (a, b).

(i) (a, b) $\neq$ (b, a)

(ii) If (a, b) = (c, d) then a = c and b = d.

RELATIONS AND FUNCTIONS

1. **Relation :** A relation R from a set A to a set B is a set of order pairs (x, y) such that $x \in A$ and $y \in B$ and x is related to y.

$\therefore R = \{x, y) : x \in A,\ y \in B \text{ and } xRy\}$

where $xRy \Rightarrow x$ is related to y. Here y is called the image of x.

Example

Let A = {2, 3, 4}, B = {6, 7, 8}

$\therefore R = \{x, y) : x \in A,\ y \in B \text{ and } xRy\}$

where $xRy \Rightarrow x$ is a factor of y

= {(2, 6), (2, 8), (3, 6), (4, 8)}

Here we see that $R \subset A \times B$.

Note -

1. A relation R from A to B is defined as a subset of A x B.
2. $xRy \Rightarrow (x, y) \in R$
3. Since $\varnothing \subset A \times B$, so $\varnothing$ is a relation from A to B.
4. Since $A \times B \subset A \times B$, so A x B is a relation from A to B.

Domain : The set of all first elements of the order pairs in R is called the domain of the relation.

$\therefore$ Domain $R \subset A$.

Codomain : If R be a relation from A to B then the Set B is called the Codomain of the relation R.

Codomain of R = B.

Range : The set of second elements in the relation R from A to B is called the range.

Range of R $\subset$ Codomain.

2. **Inverse Relation :** If R be a relation from A to B then its inverse relation R^{-1} from B to A a defined as $R^{-1} = \{(y, x) : (x, y) \in R\}$.

Example :

If R = {(2, 6), (2, 8), (3, 6), (4, 8)}

then R-1 = {(6, 2), 8, 2), (6, 3), (8, 4)}

Note : If R be a relation from A to B then

(i) Domain R = Range (R^{-1})

(ii) Range R = Domain (R^{-1})

3. **Different Types of Relations**

Relations from A to B are of four types.

(i) One - one relation

(ii) Many -one relation

(iii) One - many relation

(iv) Many - many relation

(i) **One - one relation :** A relation R from A to B is said to be one - one if different elements of A have different images in B.

Turn for $x_1, x_2 \in A$ and

$x_1 \neq x_2 \Rightarrow y_1 \neq y_2$

(ii) **Many - one relation :** A relation R from A to B is said to be many - one if different elements of A have the same image is B.

$\therefore$ For $(x_1, y_1) \in R$ and $(x_2, y_2) \in R$ and

$x_1 \neq x_2$ but $y_1 = y_2$.

(iv) **Many - many relation :** A relation R form A to B is said to be many - many if it is many - one and one - many.

Note : If A = {a, b, c}, then the smallest relation on A is $\varnothing$ and the largest relation on A is A x A.

4. **Functions** : A relation f from a set A to a set B is said to be a function if for every $x \in A$, there exist one and only one image $y \in B$ where $(x, y) \in f$.

Here y is called the image of x and is written as y = f(x).

x is called the pre-image.

A is the called the domain and B is called the Codomain.

The set of all images is called the range and is denoted by f(A).

$\therefore f(A) = \{f(x) : x \in A\}$

Note :

(1) Range is a subset of Codomain i.e. $f(a) \subset B$.

(2) If f is a function from A to B and $(x, y) \in f$ then f(x)=y.

(3) A function f from A to B is denoted by $f : A \to B$.

5. **Equal Functions :** Two functions $f : A \to B$ and $g : A \to B$ are said to be equal if for every $x \in A,\ f(x) = g(x)$.

Thus two functions f and g are equal and is written as f = g if

(i) domain f = domain g

(ii) range f = range g

(iii) for every $x \in$ dom f or dom g f(x) = g(x).

6. **Real valued function :** A function whose range is either the set R or one of its subsets is called the real valued function.

If the domain of each function is either R or a subset of R then it is called a real function.

7. **Constant function :** A function $f : R \to R$ defined by y = f(x) = c for every $x \in R$ is called a constant function.

Range of this constant function is {c}.

8. **Identity function :** A function $f : R \to R$ defined by for each $x \in R$, f(x) = x is called the identity function.

9. **Absolute value function (Modules function):**

A function $f : R \to R$ defined by $y = f(x) = |x|$

where $|x| = \begin{cases} x \text{ when } x \geq o \\ -x \text{ when } x < 0 \end{cases}$

is called an absolute value function or modules function.

10. **Signum function :** A function $f : R \to R$ is called a signum function if for each $x \in R$, $f(x) = \text{Sgn}(x)$.

Where $\text{Sgn}(x) = \begin{cases} \dfrac{|x|}{x} \text{ when } x \neq 0 \\ 0 \text{ when } x = 0 \end{cases}$

Above function can be written as

$$\text{Sgn}(x) = \begin{cases} -1 \text{ when } x < 0 \\ 0 \text{ when } x = 0 \\ 1 \text{ when } x > 0 \end{cases}$$

11. **Greatest integer function (Bracket function):** A function $f : R \to R$ is called the greatest integer function if for each $x \in R$.

f(x) = [x] where [x] is the greatest integer less than or equal to x.

$$\text{Thus } [x] = \begin{cases} -2 \text{ when } -2 \le x < -1 \\ -1 \text{ when } -1 \le x < 0 \\ 0 \text{ when } 0 \le x < 1 \\ 1 \text{ when } 1 \le x < 2 \\ 2 \text{ when } 2 \le x < 3 \end{cases}$$

12. **Exponetial function :** A function $f : R \to R$ is called an exponential function if for each $x \in R$, $f(x) = a^x$ where $a \in R^+$.

Laws of Exponential Functions

For a, b > 0 and $x, y \in R$

(i) $a^x . a^y = a^{x+y}$

(ii) $\dfrac{a^x}{a^y} = a^{x-y}$

(iii) $(ab)^x = a^x.b^x$

(iv) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

(v) $(a^x)^y = a^{xy}$

(vi) If $a \ne 1, a^x = a^y \Rightarrow x = y$

(vii) If a > 1, then $a^x > a^y \Rightarrow x > y$

(viii) If a < 1 then $a^x > a^y \Rightarrow x < y$

TRIGONOMETRIC FUNCTIONS

1. **Measure of an angle :** The measure of an angle is the amount of rotation from the initial side to the terminal side.

The angle is positive if the direction of the rotation is anticlockwise. The angle is negative if the direction of the rotation is clockwise.

2. **Right angle :** If the revolving ray rotates from initial position to terminal position describes one quarter of a circle, then we say the measure of the angle is a right-angle.

3. **System of measurement of angles :**

(i) **Degree measure :** If a right angle is divided in to 90 equal parts, then each part is called a degree. It is written as 1^0.

A degree divided into 60 minutes and a minute is divided in to 60 seconds.

$\therefore$ 1 right angle = 90^0

$1^0 = 60'$

$1' = 60''$

(ii) **Radian measure :** The angle subtended at the centre of a circle by an arc of length equal to the radius of the circle is called 1 radian. It is denoted by 1^c.

(iii) **Relation between degree and radian**

2π radian = 360^0

$\Rightarrow \pi$ radian = 180^0

$\dfrac{\pi}{2}$ radian = 90^0

1 radian = $\dfrac{180^0}{\pi}$ degree.

1 degree = $\dfrac{\pi}{180^0}$ radian.

4. **Trigonometric functions of compound angles**

$\text{Sin}(x+y) = \text{Sin } x \text{ Cos } y + \text{Cos } x \text{ Sin } y$

$\text{Sin}(x-y) = \text{Sin } x \text{ Cos } y - \text{Cos } x \text{ Sin } y$

$\text{Cos}(x+y) = \text{Cos } x \text{ Cos } y - \text{Sin } x \text{ Sin } y$

$\text{Cos}(x-y) = \text{Cos } x \text{ Cos } y + \text{Sin } x \text{ Sin } y$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\text{Cot}(x+y)=\frac{\text{Cot}\,x\,\text{Cot}\,y-1}{\text{Cot}\,y+\text{Cot}\,x}$$

$$\text{Cot}(x-y)=\frac{\text{Cot}\,x\,\text{Cot}\,y+1}{\text{Cot}\,y-\text{Cot}\,x}$$

$$\tan(x+y+z)=\frac{\tan x+\tan y+\tan z-\tan x\tan y\tan z}{1-\tan x\tan y-\tan y\tan z-\tan z\tan x}$$

5. **Product of Sine and Cosine functions as their sum and difference**

2 Sin x Cos y = Sin (x+y) + Sin (x-y)

2 Cos x Sin y = Sin (x+y) - Sin (x-y)

2 Cos x Cos y = Cos (x+y) + Cos (x-y)

2 Sin x Sin y = Cos (x-y) - Cos (x+y)

6. **Sum or difference of Sine and Cosine**

$$\text{Sin}\,C+\text{Sin}\,D=2\,\text{Sin}\frac{C+D}{2}\text{Cos}\frac{C-D}{2}$$

$$\text{Sin}\,C-\text{Sin}\,D=2\,\text{Cos}\frac{C+D}{2}\,\text{Sin}\frac{C-D}{2}$$

$$\text{Cos}\,C+\text{Cos}\,D=2\,\text{Cos}\frac{C+D}{2}\,\text{Cos}\frac{C-D}{2}$$

$$\text{Cos}\,C-\text{Cos}\,D=-2\,\text{Sin}\frac{C+D}{2}\,\text{Sin}\frac{C-D}{2}$$

$$=2\,\text{Sin}\frac{C+D}{2}\text{Sin}\frac{D-C}{2}$$

7. **Multiple angle formula**

$$\text{Sin}\,2x=2\,\text{Sin}\,x\,\text{Cos}\,x=\frac{2\tan x}{1+\tan^2 x}$$

$$\text{Cos}\,2x=\text{Cos}^2x-\text{Sin}^2x$$

$$=2\text{Cos}^2x-1=1-2\,\text{Sin}^2x$$

$$=\frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$$

$$\text{Cot}\,2x=\frac{\text{Cot}^2x-1}{2\,\text{Cot}\,x}$$

$$2\,\text{Cos}^2x=1+\text{Cos}\,2x$$

$$2\,\text{Sin}^2x=1-\text{Cos}\,2x$$

$$\text{Sin}\,3x=3\,\text{Sin}\,x-4\,\text{Sin}^3x$$

$$\text{Cos}\,3x=4\,\text{Cos}^3x-3\,\text{Cos}\,x$$

$$\tan 3x=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$$

8. **Submultiple angle formula**

$$\text{Sin}\,\theta=2\,\text{Sin}\frac{\theta}{2}\text{Cos}\frac{\theta}{2}=\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$\text{Cos}\,\theta=\text{Cos}^2\frac{\theta}{2}-\text{Sin}^2\frac{\theta}{2}=2\text{Cos}^2\frac{\theta}{2}-1$$

$$=1-2\,\text{Sin}^2\frac{\theta}{2}=\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$\tan\theta=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}},\quad \text{Cot}\,\theta=\frac{\text{Cos}^2\frac{\theta}{2}-1}{2\,\text{Cot}\frac{\theta}{2}}$$

9. **Maximum and minimum values of $a\,\text{Cos}\,\theta+b\,\text{Sin}\,\theta$**

The maximum value of $a\,\text{Cos}\,\theta+b\,\text{Sin}\,\theta$ is $\sqrt{a^2+b^2}$

The minimum value of $a\,\text{Cos}\theta+b\text{Sin}\theta$ is $-\sqrt{a^2+b^2}$

i.e. $-\sqrt{a^2+b^2}\le a\,\text{Cos}\,\theta+b\,\text{Sin}\,\theta\le\sqrt{a^2+b^2}$

10. **Trigonometric Equations**

(i) If $\text{Sin}\,\theta=0$, then $\theta=n\pi$ where $n\in Z$.

(ii) If $\text{Cos}\,\theta=0$, then $\theta=(2n+1)\frac{\pi}{2}$ where $n\in Z$.

(iii) If $\text{Sin}\,\theta = \text{Sin}\,\alpha$ where α lies between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, then $\theta = n\pi + (-1)^n \alpha$ where $n \in Z$.

(iv) If $\text{Cos}\,\theta = \text{Sin}\alpha$ where α lies between 0 and π, then $\theta = 2n\pi \pm \alpha$ where $n \in Z$.

(v) If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$ where $n \in Z$.

11. **Applications of Sine and Cosine formulae**

(i) **Sine formula :** In any triangle ABC, $\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}} = 2R$ where R is the radius of the circle circumscribing the trianlge ABC.

(ii) **Cosine formula :** In any triangle ABC,

$$a^2 = b^2 + c^2 - 2bc\,\text{Cos}A$$

$$b^2 = c^2 + a^2 - 2ca\,\text{Cos}\,B$$

$$c^2 = a^2 + b^2 - 2ab\,\text{Cos}\,C.$$

$$\therefore \text{Cos}\,A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{Cos}\,B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\text{Cos}\,C = \frac{a^2 + b^2 - c^2}{2ab}.$$

(iii) **Projection formula**

In any triangle ABC,

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\text{Cot}\frac{A}{2}$$

$$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\text{Cot}\frac{B}{2}$$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\text{Cot}\frac{C}{2}.$$

(v) **Semi-angle formula**

In triangle ABC

$$\text{Sin}\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\text{Sin}\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

$$\text{Sin}\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\text{Cos}\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\text{Cos}\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$$

$$\text{Cos}\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

12. **Principle of Mathematical Induction**

If P(n) be a statement such that

(i) P(1) is true

(ii) P(k) is true $\Rightarrow$ P(k+1) is true for all poscitive integer k, then P(n) is true for all positive integer n.

13. **Complex numbers and Quadratic equations**

(i) **Complex number :** A number of the form a+ib where $a, b \in R$ and $i = \sqrt{-1}$ is called a complex number. It is denoted by z

$\therefore Z = a + ib$.

The set of complex numbers is usually denoted by C.

$\therefore C = \{Z = a + cb : a, b \in R\}$.

(ii) **Modulus and Agrument of a Complex Number**

If $Z = a + ib$ be a complex number then its modulus is defined as $\sqrt{a^2 + b^2}$.

It is denoted by $|Z|$

$\therefore\ |Z| = \sqrt{a^2 + b^2}$.

$\tan^{-1}\dfrac{b}{a}$ is called the argument or amplitude of a complex number. It is written as Arg (Z) or Amp (Z).

$\therefore |Z| = \sqrt{a^2 + b^2}\ \&\ \text{Arg}(Z) = \tan^{-1}\dfrac{b}{a}$.

(iii) **Conjugate of a complex number :**

If two complex numbers differ only in the sign of the imaginary parts, then one is called the conjugate of the other. If Z be a complex number, then its conjugate is denoted by $\overline{Z}$.

$\therefore$ If $Z = a + cb$, then $\overline{Z} = a - ib$.

14. **Rules in the complex number system.**

(i) $\overline{Z_1 + Z_2} = \overline{Z_1} + \overline{Z_2}$

(ii) $\overline{Z_1 . Z_2} = \overline{Z_1} . \overline{Z_2}$

(iii) $\overline{\left(\dfrac{Z_1}{Z_2}\right)} = \dfrac{\overline{Z_1}}{\overline{Z_2}}$

(iv) $|Z_1 Z_2| = |Z_1|\,|Z_2|$

(v) $\left|\dfrac{Z_1}{Z_2}\right| = \dfrac{|Z_1|}{|Z_1|}$

(vi) $|Z_1 + Z_2| \le |Z_1| + |Z_2|$

15. **Cube roots of unity**

(i) $\sqrt[3]{1} = 1, \dfrac{-1+\sqrt{-3}}{2}, \dfrac{-1-\sqrt{-3}}{2} = 1, \omega, \omega^2$.

(ii) $\omega^3 = 1,\ 1 + \omega + \omega^2 = 0$

16. **Linear inequalities :**

The relations $ax + by \le c$, $ax + by \ge c$, $ax + by < c$ and $ax + by > c$ are called inequalities.

17. **Permutations and Combinations**

(i) **Fundamental Principle of Counting :** If an event occurs m different ways and corresponding to each occurance of this event another event occur in n different ways, then the total number of occurance of the two events = m x n.

Again corresponding to each occurance of the two events, if a third even occurs in p number of ways, then the total number of occurance of the three events = m x n x p.

(ii) **Permutation :** An arrangement of a number of objects taken some or all at a time in a definite order is called permutation.

The number of permutations of n different things taken r - at a time is written as n_{p_r} or p (n, r) where $r \le n$.

n_{p_r} or p (n, r)

= n (n - 1) (n - 2) ... to r factors

= n (n - 1) (n - 2) ... [n - (r - 1)]

= n (n - 1) (n - 2) ... (n - r + 1).

Also n_{p_r} or $P(n, r) = \dfrac{n!}{(n-r)!}$

Note : $n_{p_n} = n!$

(iii) **Permutations when objects are repeated :** The number of permutations of n different things taken r - at a time where each thing is repeated upto r lines is n^r.

(iv) **Permutations when all objects are not distinct :** The number of permulation of n objects where p number of objects are of one kind q number of objects are of another kind and r number of objects are of the third kind and rest are different taken all at time $= \dfrac{n!}{p!q!r!}$

(v) **Circular Permutations :** The number of circular permutations of n different objects is (n-1) !

(vi) **Combination :** An arrangement of different objects obtained by taking some or all at a time where order of the selection is not taken in to consideration is called combination.

The number of combinations of n different objects taken r - at a time is

$$c(n,r) = \frac{n(n-1)(n-2)...(n-r+1)}{r!}$$

$$= \frac{n!}{r!(n-r)!}$$

(vii) **Complementary Combinations**

c (n, r) = c (n, n - r),

Rules

(a) c (n, r) + c(n, r - 1) = c (n +1, r)

(b) $\frac{c(n,r)}{c(n,r-1)} = \frac{n-r+1}{r}$

(c) ${}^nc_1 + {}^nc_2 + {}^nc_3 + ... + {}^nc_n = 2^n - 1$

18. **Binomial theorem :**

(i) If n is a positive integer, then

$$(a+b)^n = n_{c_0} a^n + n_{c_1} a^{n-1}b +$$

$$n_{c_2} a^{n-2}b^2 + ... + n_{c_n} b^n$$

(ii) General term

$$n_{c_r} a^{n-r}b^r = \frac{n(n-1)(n-2)...(n-r+1)}{r!} a^{n-r}b^r$$

(iii) If a = 1, and b = a, then

$$(1+a)^n = n_{c_0} + n_{c_1}a + n_{c_2}a^2 + ... + n_{c_n}a^n$$

$n_{c_0}, n_{c_1}, n_{c_2}, ..., +n_{c_n}$ are binomial coefficients. They are written as

$c_0, c_1, c_2, ..., c_n$.

$$\therefore\ (1+a)^n = c_0 + c_1a + c_2a^2 + ... + c_na^n .$$

(iv) $c_0 + c_1 + c_2 + ... + c_n = 2^n$

(v) $c_0 + c_2 + c_4 + ... = c_1 + c_3 + c_5 + ... = 2^{n-1}$

19. **Sequence and Series**

(i) **Sequence :** A set of numbers arranged in a definite order according to some rule is called a sequence.

(ii) **Arithmetic Progression (A.P.) :** A sequence where the differene of a term with the previous term is always same is called arithmetic progression. This difference is called the common difference.

If a be the first term and d be the common difference then nth term = a + (n - *1*) d.

S_n = Sum of n terms $= \frac{n}{2}$ [2a + (n - *1*) d].

If *l* be the nth term, then *l* = a + (n - *1*) d.

$$\therefore S_n = \frac{n}{2}(a+l).$$

(iii) **Arithmetic Mean :** The arithmetic mean of a and b is $\frac{a+b}{2}$.

The n number of arithmetic mean between a and b are

$$a + \frac{b-a}{n+1},\ a + \frac{2(b-a)}{n+1},$$

$$a + \frac{3(b-a)}{n+1}, ..., a + \frac{n(b-a)}{n+1}.$$

(iv) **Geometric Progression (G.P.) :** A sequence of numbers is called a geometric progression if the ratio of a term with the previous term is always a constant.

This constant ratio is called the common ratio (c.r.).

If a, b, c, d, e, ... be is G. P.

then $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{e}{d} = ...$

If a be the first term and r be the common ratio then nth term = a . r^{n-1} i.e. $t_n = a\, r^{n-1}$.

If S_n be the sum of n terms of a G.P. then

$$S_n = \frac{a(1-r^n)}{1-r} \text{ if } |r| < 1$$

and $S_n = \frac{a(r^n - 1)}{r-1}$ if $|r| > 1$.

(v) **Geometric Mean :** The geometric mean of two numbers a and b is $\sqrt{ab}$.

(vi) **Arithmetico-Geometric Sequence :**

The sequence a, (a + d) r, (a + 2d) r^2, ..., [a + (n - 1)d]r^{n-1}, ... is called arithmetico geometric sequence.

(vii) **Harmonic Progression (H.P.) :** A sequence of terms a, b, c, d, ... are in Harmonic progresson if $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \ldots$ are in arithmetic progression.

(viii) **Harmonic Mean (H.M.) :** The harmonic mean of a and b is $H = \frac{2ab}{a+b}$.

(ix) **Relation between A.M., G.M. and H.M. of two positive real numbers :** Let a and b are two given positive real numbers. Let A, G and H be the A.M., G.M. and H.M. of a and b respectively.

$$A = \frac{a+b}{2},\ G = \sqrt{ab},\ H = \frac{2ab}{a+b}$$

$\therefore A \geq G \geq H$.

20. **Exponential Series**

(i) The exponential series e is defined as

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$$

$$\frac{e^x - e^{-x}}{2} = x - \frac{x^3}{3!} + \frac{x^5}{4!} + \ldots$$

(ii) **Expansion of a^x where $a \in R$.**

$$a^x = 1 + \frac{x}{1!}(\log_e a) + \frac{x^2}{2!}(\log_e a)^2 + \ldots$$

21. **Logarithmic Series**

If $|x| < 1$,

then $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

22. **Straight line**

(i) **Slope of a line :** If a straight line makes an angle θ with the positive direction of x - axis, then the slope of the line is $\tan\theta$. It is denoted by m. $m = \tan\theta$.

(ii) Slope of a line passing through points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(iii) **Angle between two lines :** The angle θ between two lines whose slopes are m_1 and m_2 is given by $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

where m_1 & m_2 are the slopes of initial and terminal sides.

Notes :

(a) If two lines are parallel then $m_1 = m_2$.

(b) It two lines are perpendicular then $m_1 m_2 = -1$.

(iv) **Locus :** The set of points satisfying a geometric condition is called a locus.

Examples : Straight line, Circle, Parabala, Ellipse, Hyperbola.

23. **Equation of a straight line**

(i) **Slope point form :** Equation a line passing through a point (x_1, y_2) and whose slope is m is $y - y_1 = m(x - x_1)$.

(ii) **Slope intercept form :** The equation of a line whose slope is m and whose y - intercept is c is y = mx + c.

(iii) **Two points form :** Equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

(iv) **Intercepts form :** Equation of a line whose x - intercept is a and y - intercept b is $\frac{x}{a}+\frac{y}{b}=1$.

(v) **Perpendicular form :** If p be the length of the perpendicular from the origin to a straight line and α be the inclination of this perpendicular then the equation of the line is $x\cos\alpha + y\sin\alpha = p$.

(vi) **Parametric form :** The equation of a line passing through a point (x_1, y_1) and making an angle θ with x - axis is $\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r$.

(vii) **General form :** The general equation of the line is ax + by + c = 0.

Its slope is m = $-\frac{b}{a}$

Its x - intercept = $-\frac{c}{a}$

Its y - intercept = $-\frac{c}{b}$.

(viii) The length of the perpendicular from a point (x_1, y_1) to the line ax + by + c = 0 is $p=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$.

(ix) **Bisectors :** The equation of the bisectors of the angles between two lines $a_1x+b_1y+c_1=0$ and $a_2x + b_2y + c_2 = 0$ is

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}.$$

24. **Circle :** A set of points on a plane which is equidistant from a fixed point is called a circle.

The fixed point is called centre and the constant distance is called radius.

(i) **Centre-radius form :** The equation of a circle whose cenre is (h, k) and whose radius is r is $(x-h)^2+(y-k)^2 = r^2$.

(ii) **General form :** The general form of the equation of a circle is $x^2+y^2+2gx+2fy+c=0$.

Its centre is (-g, -f) and radius $\sqrt{g^2+f^2-c}$.

25. **Parabola :** A set of points equidistant from a fixed point and a fixed line is called the parabola.

The fixed point is called **focus** and the fixed line is called the **directrix**. The line passing through the focus and perpendicular to the directrix is called **axis**. The point where the axis intersect the parabola is called **vertex**.

(i) The equation of a parabola whose vertex is (0, 0) and whose focus is (0, 0) is $y^2 = 4ax$.

Here (i) Vertex is (0, 0).

(ii) Focus is (a, 0)

(iii) Equation of the directrix is x = -a

(iv) Equation of the latus rectum is x=a.

(v) Length of the latus rectum = 4a.

(vi) If a is +ve, the parabola opens to the right and if a is -ve, the parabola opens to the left.

26. **Ellipse :**

(i) A set of points on a plane is defined as an ellipse if the sum of its distances from two fixed points on a plane is constant.

Each fixed point is called the focus.

The line segment passing known two foci and is intercepted by the ellipse is called the major axis.

The line segment passing through the centre and perpendicular to the major axis and intercepted by the ellipse is called the minor axis.

(ii) An ellipse is the set of all points in the plane whose distance from a fixed point bears a constant ratio with its distance from a fixed line and this constant ratio is less than 1.

The fixed point is called the focus and the fixed line is called the directrix. The constant ratio is called the eccentricity and is denoted by e.

27. **Equation of the ellipse**

The equation of the ellipse whose centre is (0, 0) and major axis is along x - axis and minor axis is along y - axis is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Where $b^2 = a^2 - c^2$.

Here (i) Centre is (o, o)

(ii) The coordinates of the foci are ($\pm$ c, o)

(iii) Length of the major axis is 2a. End points of the major axis are ($\pm$ a, o)

(iv) Length of the minor axis is 2b. End points of the minor axis are (o, $\pm$ b).

(v) End points of the latus rectum are $\left(\pm c, \pm\frac{b^2}{a}\right)$.

(vi) The equation of the directrix are $x=\pm\frac{a^2}{c}$.

28. **Hyperbola**

A hyperbola is the locus of a point in a plane which moves such that the difference of its distances from two fixed points in a plane is constant.

Each fixed point is called the focus.

A hyperbola is the locus of a point in a plane which moves such that its distance from a fixed point bears a constant ratio with its distance from a fixed line and this constant ratio is greater than 1.

The fixed point is called the focus and the fixed line is called the directrix. The constant ratio is called the eccentricity e and e >1.

29. **Equation of the hyperbola**

Equation of the hyperbola whose centre is (o, o) and the transverse axis is along x-axis, is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

where $b^2 = c^2 - a^2$.

(i) The coordinates of the centre is (o, o).

(ii) The coordinates of foci are ($\pm$ c, o)

(iii) The length of the transverse axes = 2a.

End points of the transverse axis are ($\pm$ a,o).

(iv) The length of the conjugate axes = 2b.

End points of the conjugate axis are (o, $\pm$ b).

(v) The length of the latus rectum $\frac{2b^2}{a}$.

End points of the latus rectum are $\left(\pm c, \pm\frac{b^2}{a}\right)$.

30. **Three dimensional Geometry**

(i) **Distance formula :** The distance between two points $p(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$PQ\, PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

(ii) **Section formula :** The coordinates of a point which divides the line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio m:n is given by

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$$

(iii) **Middle point :** The coordinates of the middle point of the line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

(iv) **Direction cosines of a line :** The cosines of the angles which a line makes with the positive direction of x-axis, y-axis and z-axis are called the direction cosines (d.c.) of the line.

If a line makes the angles α, β, γ with the positive direction of x-axis, y-axis and z-axis, then $\cos\alpha, \cos\beta, \cos\gamma$ are called the direction cosines of the line. They are denoted by l, m, n.

$\therefore l=\cos\alpha,\ m=\cos\beta,\ n=\cos\gamma$

Note $l^2 + m^2 + n^2 = 1$.

(v) **Direction ratios of a line :** Any three numbers which are proportional to the direction cosines of the line are called the direction ratios of the line.

If, l, m, n be the direction cosines and a, b, c be the direction ratios of the line, then

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c}=\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}=\frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}},$$

$$m=\frac{b}{\sqrt{a^2+b^2+c^2}},$$

$$n=\frac{c}{\sqrt{a^2+b^2+c^2}}.$$

31. **Limits and Derivatives**

(i) **Limit :** If a function f(x) approches l when x tends to a, then l is called the limit of the function.

It is written as $\lim_{x\to a} f(x)=l$.

$\Leftrightarrow f(x)\to l$ when $x\to a$.

(ii) **Left hand limit :** When a function f(x) approches l when x tends to a from left, then l is called the left hand limit.

It is written as $\lim_{x\to a^-} f(x)=l$

$\Leftrightarrow \lim_{h\to 0} f(a-h)=l$.

(iii) **Right hand limit :** When a function f(x) approches l when x tends to a from right, then l is called the right hand limit.

It is written as $\lim_{x\to a^+} f(x)=l$

$\Leftrightarrow \lim_{h\to 0} f(a+h)=l$

Note :

(i) If $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)=l$ then the limit of the function exists.

(ii) If $\lim_{x\to a^-} f(x)\neq\lim_{x\to a^+} f(x)$, then the limit of the function does not exist.

(iii) $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\ (n\in N)$

(iv) $\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)=1$

(v) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$

(vi) $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$

(vii) $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e$

(viii) $\lim_{x\to 0}\frac{e^x-1}{x}=1$

32.(i) **Derivative of a function at a point :** A a fuction 'f' is said to be differentiable at x = a if $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ exists. This limit is called the derivative of the function at x = a.

It is denoted by $f^1(a)$ or $\left[\frac{df(x)}{dx}\right]_{x=a}$.

(ii) **Left hand dervative :** A function 'f' is said to be differentiable at x = a from left if $\lim_{x\to a^-}\frac{f(x)-f(a)}{x-a}$ exists.

It is denoted by $f^1(a^-)$.

$\therefore f^1(a^-)\ \lim_{x\to a^-}\frac{f(x)-f(a)}{x-a}$

$=\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$.

(iii) **Right hand derivative :** A function f is said to be differentiable at x = a from right if

$$\lim_{x\to a^+}\frac{f(x)-f(a)}{x-a} \text{ exists.}$$

It is denoted by $f^1(a^+)$.

$$\therefore f^1(a^+) = \lim_{x\to a^+}\frac{f(x)-f(a)}{x-a}$$

$$= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Notes :

(i) If $f^1(a^-) = f^1(a^+)$ then the derivative of the function exists at x = a.

(ii) If $f^1(a^-) \neq f^1(a^+)$ then the derivative of the function does not exist at x = a.

33. **Derivative of some functions.**

(i) $\dfrac{dx^n}{dx} = nx^{n-1}$

(ii) $\dfrac{d\,\text{Sin}\,x}{dx} = \text{Cos}\,x$

(iii) $\dfrac{d\,\text{Cos}\,x}{dx} = -\text{Sin}\,x$

(iv) $\dfrac{d\tan x}{dx} = -\text{Sec}^2 x$

(v) $\dfrac{d\,\text{Cot}\,x}{dx} = -\text{Co}\sec^2 x$

(vi) $\dfrac{d\,\text{Sec}\,x}{dx} = \text{Sec}\,x\tan x$

(vii) $\dfrac{d\,\text{Co}\sec x}{dx} = -\text{Co}\sec x\,\text{Cot}\,x$

33. **Mathematical Reasoning**

(i) **Statement :** A statement is a declarative sentence which is either true or false but not both. A statement is denoted by letters p, q, r, ...

(ii) **Negation of a statement :** The denial of a statement is called its negation. If p be a statement, its negation is denoted by ~p.

(iii) **Conjunction :** If two statements are connected by the word "and", then the statement so formed is called a conjuction.

If p, q be two statements, then the conjunction is denoted by $p \wedge q$.

Note : A conjunction is true when all the component statements are true otherwise it is false.

(iii) **Disjunction :** It two statements are connected by the word 'or' then the statement so formed is called disjunction. If p, q be two statements, then the disjunction is p or q. It is denoted by $p \vee q$.

Note : A disjunction is false when both the connecting statements are false otherwise it is true.

(iv) **Conditional (Implication) :** If p, q be two statemens then "if p then q" is called complication. It is denoted by $p \to q$.

Here p is called the antecendent and q is called the consequent.

Note : A conditional is false if the antecendent is true and he consequent is false otherwise it is true.

(v) **Converse :** In a conditional if the antecendent is changed to consequent and vice verse, then the statement so formed is called the converse of the conditional.

If $p \to q$ is a conditional, then its converse is $q \to p$.

(vi) **Biconditional (Double implication) :** The conjunction of a conditional and and its converse is called biconditional. If p, q below statements, thn the biconditional is $(p \to q) \wedge (q \to p)$. It is written as $p \leftrightarrow q$.

Note : A biconditional is true when the antecedent and consequent are both true or both false otherwise it is false.

(vii) **Inverse :** If $p \to q$ be conditional then its inverse is defined as $\sim p \to \sim q$.

(viii) **Contrapositive :** The inverse of the converse of a conditional is called its contrapositive.

Thus the contrapositive of $p \to q$ is $\sim q \to \sim p$.

(ix) **Equivalent Statments :** Two statements are said to be logrcally equivalent of they have the same truth values.

(x) **Tautology :** If a compound statement is always true then it is called a tautology. If p be a statement then $p \vee \sim p$ is atautology.

(xi) **Contradiction :** If a compound statement is always false, then it is contradiction. If p be a statement then $p \wedge \sim p$ is a contradiction.

(xii) **De Morgan's Law :** If p, q be two statements then

$$\sim (p \vee q) = \sim p \wedge \sim q$$

$$\sim (p \wedge q) = \sim p \vee \sim q$$

35. **Statistics**

(i) **Mean Deviation :** If $x_1, x_2 \dots x_n$ be n variables and $\bar{x}$ be their arithmetic mean, then the mean deviaiton (M.D.) is defined by

$$\text{M.D.} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \dots + |x_n - \bar{x}|}{n}$$

$$= \frac{\sum |x_i - \bar{x}|}{n}$$

If $x_1, x_2, \dots x_n$ be the variables with the corresponding frequencies $f_1, f_2, \dots, f_n$ and $\bar{x}$ be their arithmetic mean, then

Mean Deviation (M.D.) =

$$\frac{f_1|x_1 - \bar{x}| + f_2|x_2 - \bar{x}| + \dots + f_n|x_n - \bar{x}|}{f_1 + f_2 + \dots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i}$$

For a grouped frequences distribution, if $x_1, x_2, \dots, x_n$ be the midvalues with the corresponding frequencies $f_1, f_2, \dots, f_n$ and $\bar{x}$ be their arithmeti mean, then the mean deviation is

$$\text{M.D.} = \frac{f_1|x_1 - \bar{x}| + f_2|x_2 - \bar{x}| + \dots + f_n|x_n - \bar{x}|}{f_1 + f_2 + \dots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i}$$

(ii) **Standard Deviation :** For a simple distribution if $x_1, x_2, \dots, x_n$ be the n variables and $\bar{x}$ be the arithmetic mean then the standard deviation σ is defined as

$$\text{S.D.} = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}}$$

$$\Rightarrow \sigma = \sqrt{\frac{\sum_{i-1}^{n} (x_i - \bar{x})^2}{n}}$$

For a frequency distribution if $x_1, x_2, \dots x_n$ be the n variables with the corresponding frequencies $f_1, f_2, \dots, f_n$ and $\bar{x}$ be the arithmetic mean then the standard deviation is defined as

$$\text{S.D.} = \sqrt{\frac{f_1(x_1-\bar{x})^2 + f_2(x_2-\bar{x})^2 + f_n(x_n-\bar{x})^2}{f_1+f_2+...+f_n}}$$

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{f_i(x_i-\bar{x})^2}{\sum_{i=1}^{n} f_i}}$$

For a grouped frequency distribution, if $x_1, x_2, ... x_n$ are the midvalues of the class intervals with frequencies $f_1, f_2, ..., f_n$ and $\bar{x}$ be the arithmetic mean, then the standard deviation is defined as

$$\text{S.D.} = \sqrt{\frac{f_1(x_1-\bar{x})^2 + f_2(x_2-\bar{x})^2 + f_n(x_n-\bar{x})^2}{f_1+f_2+...+f_n}}$$

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{f_i(x_i-\bar{x})^2}{\sum_{i=1}^{n} f_i}}$$

(iii) **Variance :** The square of the standard deviation is known as variance. It is denoted by υ.

$\therefore \upsilon = \sigma^2$.

Rules : For a frequency distribution

$$\upsilon = \sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} = \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2.$$

(iv) **Coefficient of variation :** Coefficient of Variation (C.V.) is defined as $\text{C.V.} = \frac{\sigma}{\bar{x}} \text{x} 100$ where σ is the standard deviation.

36. **Probability**

(i) If A be an event and S be the sample space, then the probability of A i.e. P(A) is defined as $P(A) = \frac{|A|}{|S|}$.

Rule :

1. $0 \le P(A) \le 1$.
2. If A and B are two events, then $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive.
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
4. If A, B and C are three events then $P(A \cup B \cup C) = P(A) + P(B) - P(A \cap B)$ $- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

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GROUP - A

OBJECTIVE AND VERY SHORT TYPE QUESTIONS

Each questions carries 1 Mark.

1. If a set contains n elements, then what is the number of elements in its power set ?

2. If $n(A\bigcup B)=50$, $n(A)=28$ and $n(B)=32$, then what is $n(A\bigcap B)$?

3. If A and B are two disjoint sets, then what is $n(A\bigcup B)$.

4. If A and B are two sets such that $A\subset B$, then what is $A\bigcup B$?

5. If $aN=\{ax : x\in N\}$ then what is $4N\bigcap 10N$?

6. In a group of 800 people, 550 can speak Hindi and 450 can speak English, then what is the number of people speaking both Hindi and English ?

7. If $|A|=m$ and $|B|=n$, then what is $|P(AxB)|$.

8. What is the number of elements of $P(P(P(\phi)))$?

9. If R be a relation from the Set A={-2,-1,0,1,2} to Set-B = {0, 1, 4, 9} and is defined by $R=\{(a,a^2) : a\in A\}$. Find R in roster form.

10. A relation R from the Set A = {2, 3, 4, 5} to a Set B={3, 6, 7, 10} defined by $R=\{(x,y) : x\in A,\ y\in B \text{ and } x \text{ divides } y\}$. Find R^{-1}.

11. If A = {1, 2} and B = {3, 4} then what is the number of relations from A to B ?

12. If $R=\{(x,\ y) : x,y\in N, x+2y=8\}$ then what is R^{-1} ?

13. If f is a real function satisfying $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}$ for all $x\in R-\{0\}$ then what is f(x) ?

14. If $f(x)=x^3-\frac{1}{x^3}$ then what is the value of $f(x)+f\left(\frac{1}{x}\right)$?

15. What is the domain of the function $f(x)=\sqrt{a^2-x^2}$?

16. Wha is the domain of the function $f(x)=\frac{x^2+2x+3}{x^2-5x+4}$?

17. If $f(x)=(x^2-a)^2(x-b)^2$ then what is $f(a+b)$?

18. If Sin x + Cosec x = 2 then what is the value of $\text{Sin}^n x + \text{Cosec}^n x$?

19. If $A+B=\frac{\pi}{4}$ then what is the value of (tan A+1) (tan B+1) ?

20. If A, B, C, D be the angles of a cyclic quadrilateral ABCD, then what is the value of Cos A + Cos B + Cos C + Cos D ?

21. What is the value of $\tan\frac{\pi}{20}.\tan\frac{3\pi}{20}.\tan\frac{5\pi}{20}.\tan\frac{7\pi}{20}.\tan\frac{9\pi}{20}$?

22. If $A+B=\frac{\pi}{3}$ and Cos A + Cos B = 1, then what is the value of $\text{Cos}\frac{A-B}{2}$?

23. What is the value of $\text{Sin}\frac{\pi}{12}.\text{Sin}\frac{5\pi}{12}$?

24. If $\text{Sin}\,x+\text{Sin}^2x=1$ then what is the value of $\text{Cos}^2x+\text{Cos}^4x$?

25. If $A+B=\frac{\pi}{4}$ then what is the value of (Cot A-10 (Cot B-1) ?

26. What is the general solutions of $\text{Sin}\frac{\theta}{2}=-1$?

27. What is the general solution of the equation $7\text{Cos}^2\theta+3\text{Sin}^2\theta=4$?

28. If the sum of he squares of the sides of the inscribed triable ABC is equal to the twice the square of the diameter of the circle, then what is $\text{Sin}^2A+\text{Sin}^2B+\text{Sin}^2C$?

29. In a triangle ABC, if the angles A, B, C are in A.P., then what is the value of

$$\frac{a+c}{\sqrt{a^2-ac+c^2}}\ ?$$

30. If a = b Cos C then what is the value of B ?

31. What is the value of $1^2 + 2^2 + 3^2 + ... + 10^2$?

32. Choose the correct answer. For all $n \in N$, n (n+1) (n+2) is divisible by ______.

    (i) 6 (ii) 7 (iii) 8

33. What is the value of $1^3 + 2^3 + 3^3 + ... + n^3$ for $n \in N$?

34. If z = 2 - 3i then what is the value of $z^2 - 4z + 13$?

35. What is the value of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ for $n \in N$?

36. If $a+ib=\frac{c+i}{c-i}$ when c is a real number, then what is the value of $a^2 + b^2$?

37. If $\frac{1-ix}{1+ix}=a+ib$ then what is the value of $a^2 + b^2$?

38. What is the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$?

39. If $z=\frac{1+i}{1-i}$ then what is the value of z^4 ?

40. If w is the imaginary cube roots of unity then what is the value of $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$?

41. What is the smallest positive integer n for which $(1 + i)^{2n} = (1 - i)^{2n}$?

42. If ω is an imaginary cube root of unity then what is the value of

    $(2 - \omega)(2 - \omega^2)(2 - \omega^{10})(2 - \omega^{11})$?

43. If $8_{P_r}=56$ then what is the value of r ?

44. What is the number of ways in which 5 persons can sit in a round table ?

45. What is the number of ways in which one can post 5 letters in 7 letter boxes ?

46. If $n_{P_6}=20\,.\,n_{P_4}$ then what is the value n ?

47. If ${}^nc_{14}={}^nc_{16}$ then find the value of ${}^{32}c_n$.

48. Choose the correct answer.

    If $n={}^mc_2$ then the value of ${}^mc_2$ is given by

    (i) ${}^{m+1}c_4$ (ii) $2.{}^{n+1}c_4$

    (iii) $3.{}^{m+1}c_4$ (iv) $4.{}^{m+1}c_4$

49. What is the value of expression $47_{c_4} = \sum_{i=1}^{5} 52 - i_{c_3}$?

50. If $\frac{1}{4_{c_n}} = \frac{1}{5_{c_n}} + \frac{1}{6_{c_n}}$ then what is the value of n ?

51. Write the constant term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$.

52. What is the total number of term in the expansion of $(1 + x)^{10} (1 - x)^{10}$?

53. If the n^{th} term of a sequence is $a_n = (-1)^{n-1}(n+1)^2$, then find a_7.

54. If 1 + 6 + 11 + 16 + ... + x = 148, then what is x ?

55. If the ratio of the sum of m and n terms of an A.P. is $m^2 : n^2$, then what is the ratio of the m^{th} and n^{th} term ?

56. If 10 times the 10^{th} term of an A.P. is equal to 15 times 15^{th} term then what is 25^{th} term of the A.P. ?

57. If the third term of a G. P. is 4 then what is the product of the first five terms ?

58. If p^{th}, q^{th} and r^{th} term of a G. P. are x, y, z respectively then what is the value of $x^{q-r}.y^{r-p}.z^{p-q}$?

59. If A_1, A_2 be two A.M.s and G_1, G_2 be two G.M.s between a and b, then what is the value of $\frac{A_1 + A_2}{G_1 G_2}$?

60. What is the value of $\sum_{n=1}^{\infty} \frac{n^2}{n!}$?

61. If the lines 2x - 3y + 1 = 0 and 3x + ky - 1 = 0 are perpendicular to each other then what is the value of k ?

62. What is the angle between the lines x = 2 and $x - \sqrt{3}y + 1 = 0$?

63. If the line y = x + k touches the circle $x^2+y^2=16$ then what is the value of k ?

64. What is the length of the latus rectum of the parabola $(y-2)^2 = 8 (x+3)$?

65. What is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$?

66. What is the equation of the conjugate axis of the hyperbola $\frac{x^2}{9} - \frac{(y+2)^2}{16} = 1$.

67. What is the length of the projection of the line segment joining (1, 3, -1) and (3, 2, 4) on z-axis ?

68. What is the image of the point (6, 3, -4) with respect to yz - plane ?

69. What is the distance of P (a, b, c) from x - axis ?

70. If A (3, 2, -4), B (5, 4, -6) and C (9, 8, -10) are collinear then in which ratio C divides AB.

71. What is the limit of $\lim_{\theta \to \frac{\pi}{2}} \frac{\cos\theta}{\theta - \frac{\pi}{2}}$?

72. Find the limit of $\lim_{n \to \infty} \frac{1^2 + 2^2 + 3^2 + ... + n^2}{n^3}$.

73. What is the value of

$$\lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + ... + (x+100)^{10}}{x^{10} + 10^{10}} ?$$

74. Find $\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x}$.

75. Find $\lim_{x\to 0} \frac{1-\text{Cos}(1-\text{Cos}\,x)}{x^4}$.

76. Find $\lim_{x\to 0} \left(\frac{\text{Sin}\,x^0}{x}\right)$.

77. If x < 1, the what is $\frac{d}{dx}|x-1|$?

78. Find $\frac{d}{dx}[e^{a\log x}+e^{a\log a}]$.

79. If $y=\sqrt{\frac{1+\text{Cos}\,2x}{1-\text{Cos}\,2x}}$, then find $\frac{dy}{dx}$ at $x=\frac{3\pi}{4}$?

80. If $y=\text{Sec}^{-1}\left(\frac{x+1}{x-1}\right)+\text{Sin}^{-1}\left(\frac{x-1}{x+1}\right)$ then $\frac{dy}{dx}$?

81. What is the negation of $pv \sim q$?

82. The converse of $p \to \sim q$ is ?

83. What is the contrapositive of $\sim p \to \sim q$?

84. What is the man deviation of 30, 40, 85, 75 and 45 ?

85. What is the mean deviation of 30, 40, 85, 75 and 45 ?

86. Calculate the mean deviation of the following distribution.

Age in years	14	15	30
No.of persons	5	4	1

87. Define standard deviaion of a frequency distribution.

88. Define coefficient of variation.

89. Define complementary event.

90. What is the probability of getting exactly two heads in a single throw of two unbiased coins?

91. Three dice are rolled. What is the probability that the same number will appear on all the dice ?

92. Two cards are drawn from a pack of 52 cards what is the probability that both are spades ?

93. What is the probability that one digit positive integer is even ?

94. What is the probability of obtaining a total of 11 from a throw of two dice ?

95. A coin is tossed three times. Fond the probability of getting all heads.

96. Two cards are drawn from a pack of 52 cards, what is the probability that both are diamonds?

97. A die is thrown. What is the probability of getting 2 ?

98. What is the probability of getting a red card from a pack of 52 cards ?

99. A coin is tossed and then a die is thrown only in case a tail is shown on the coin. Find the sample space.

100. A coin is tossed three times, what is the probability of getting at least two tails.

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GROUP - A

ANSWERS

1. If $|A| = n$, then $|P(A)| = 2^n$.

2. We know

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 50 = 28 + 32 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 60 - 50 = 10$

3. A and B are disjoint sets.

$\therefore |A \cup B| = |A| + |B|$

$\Rightarrow n(A \cup B) = n(A) + n(B)$

4. It ACB, then $A \cup B = B$.

5. Give that

$aN = \{ax : x \in N\} = \{a, 2a, 3a, 4a, 5a, 6a, ...\}$

$4N = \{4x : x \in N\}$

$= \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...\}$

$10N = \{10x : x \in N\} = \{10, 20, 30, 40, ...\}$

$4N \cap 10N = \{20, 40, ...\}$

$= \{20x : x \in N\} = 20N$

6. Let H be the set of people speaking Hindi, E be the set of people speaking English.

$\therefore \quad |E \cup H| = 800$

$|H| = 550, |E| = 450$

$|E \cap H| = ?$

We know

$|E \cup H| = |E| + |H| - |E \cap H|$

$\Rightarrow 800 = 450 + 550 - |E \cap H|$

$\Rightarrow |E \cap H| = 1000 - 800 = 200$.

7. $|P(AxB)| = 2^{mn}$.

8. We know

$|\varnothing| = 0$

$|p(\varnothing)| = 2^0 = 1$

$|p(p(\varnothing))| = 2^1 = 2$

$|p(p(p(\varnothing)))| = 2^2 = 4$

9. Given that

$A = \{-2, -1, 0, 1, 2\},\ B = \{0, 1, 4, 9\}$

$\therefore\ R = \{(a, a^2) : a \in A\}$

$= \{(0,0), (1,1), (2,4)(-2,4), (-1,1)\}$

10. $A = \{2, 3, 4, 5\},\ B = \{3, 6, 7, 10\}$

$R = \{(x, y) : x \in A,\ y \in B \text{ and } x \text{ dividesy}\}$

$= \{(2,6), (2,10), (3,3), (3,6), (5,10)\}$

$R^{-1} = \{(6,2), (10,2), (3,3), (6,3), (10,5)\}$

11. $A = \{1, 2\},\ B = \{3, 4\}$

$A \text{ x } B = \{(1,3), (1,4), (2,3), (2,4)\}$

$|A \text{ x } B| = 4$

The number of subsets of A x B is 2^4.

The number of relations from A o B is 2^4.

12. $R = \{(x, y) : x, y \in N,\ x + 2y = 8\}$

$= \{(2,3), (4,2), (6,1)\}$

$R^{-1} = \{(3,2), (2,4), (1,6)\}$

13. Given that

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$

$= x^2 + \frac{1}{x} + 2 - 2 = \left(x + \frac{1}{x}\right)^2 - 2$

$\Rightarrow f(x) = x^2 - 2$.

14. $f(x)+f\left(\frac{1}{x}\right)=0$

15. Domain of $f(x)=\sqrt{a^2-x^2}$ is $\{x:-a\le x\le a\}$.

16. Domain of the function $f(x)=\frac{x^2+2x+3}{x^2-5x+4}$ is $R-\{1,4\}$.

17. If $f(x)=(x-a)^2(x-b)^2$ then $f(a+b)=a^2b^2$.

18. $\text{Sin}\,x+\text{Cosec}\,x=2$

$\Rightarrow \text{Sin}\,x+\frac{1}{\text{Sin}\,x}=2$

$\Rightarrow \text{Sin}^2x+1=2\text{Sin}\,x$

$\Rightarrow (\text{Sin}^2x-1)^2=0$

$\Rightarrow \text{Sin}\,x=1$

$\therefore \text{Cosec}\,x=1$

$\therefore \text{Sin}^n x+\text{Cosec}^n x=1^n+1^n=2$

19. $(\tan A+1)(\tan B+1)=2$

20. 0

21. 1

22. $A+B=\frac{\pi}{3}$

Given that $\text{Cos}\,A+\text{Cos}\,B=1$

$\Rightarrow 2\text{Cos}\frac{A+B}{2}\text{Cos}\frac{A-B}{2}=1$

$\Rightarrow 2\text{Cos}\frac{\pi}{6}\ \text{Cos}\frac{A-B}{2}=1$

$\Rightarrow 2.\frac{\sqrt{3}}{2}\ \text{Cos}\frac{A-B}{2}=1$

$\Rightarrow \text{Cos}\frac{A-B}{2}=\frac{1}{\sqrt{3}}$

23. $\text{Sin}\frac{\pi}{12}\,.\,\text{Sin}\frac{5\pi}{12}=\frac{1}{2}\,.\,2\text{Sin}\frac{\pi}{12}\,.\,\text{Sin}\frac{5\pi}{12}$

$=\frac{1}{2}\left[\text{Cos}\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)-\text{Cos}\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)\right]$

$=\frac{1}{2}\left[\text{Cos}\frac{\pi}{3}-\text{Cos}\frac{\pi}{2}\right]=\frac{1}{2}\left[\frac{1}{2}-0\right]=\frac{1}{4}$.

24. $\text{Sin}\,x+\text{Sin}^2x=1$

$\Rightarrow \text{Sin}\,x=1-\text{Sin}^2x=\text{Cos}^2x$

$\therefore \text{Cos}^2x+\text{Cos}^4x=\text{Sin}\,x+\text{Sin}^2x=1$.

25. $(\text{Cot}\,A-1)(\text{Cot}\,B-1)=2$

26. $\text{Sin}\frac{\theta}{2}=-1=\text{Sin}\left(-\frac{\pi}{2}\right)$

$\therefore \frac{\theta}{2}=n\pi+(-1)^n\left(-\frac{\pi}{2}\right)$ where $n\in Z$.

$\Rightarrow \theta=2n\pi+(-1)^{n+1}\pi$

27. $7\text{Cos}^2\theta+3\text{Sin}^2\theta=4$

$\Rightarrow 7\text{Cos}^2\theta+3(1-\text{Cos}^2\theta)=4$

$\Rightarrow 7\text{Cos}^2\theta+3-3\text{Cos}^2\theta=4$

$\Rightarrow 4\text{Cos}^2\theta=1$

$\Rightarrow \text{Cos}^2\theta=\frac{1}{4}$

$\Rightarrow \text{Cos}\,\theta=\frac{1}{2},\frac{-1}{2}$

$=\text{Cos}\frac{\pi}{3},\ \text{Cos}\frac{2\pi}{3}$

$\therefore \theta=2n\pi\pm\frac{\pi}{3}$ and $\theta=2n\pi\pm\frac{2\pi}{3}$ where $n\in Z$.

28. Diameter of the circle is 2R.

According to the question, $a^2+b^2+c^2=2.\,(2R)^2$

$\Rightarrow 4R^2(\text{Sin}^2A+\text{Sin}^2B+\text{Sin}^2C=8R^2$

$\Rightarrow \text{Sin}^2A+\text{Sin}^2B+\text{Sin}^2C=2$.

29. $\dfrac{a+c}{\sqrt{a^2-ac+c^2}}=\text{Cos}\dfrac{B-C}{2}$

30. $B = 90^0$

31. 385

32. 6

33. $\dfrac{n^2(n+1)^2}{4}$

34. Given that z = z - 3i

$\Rightarrow z-2=-3i$

$\Rightarrow (z-2)^2=-9i^2$

$\Rightarrow z^2-4z+4=-9$

$\Rightarrow z^2-4z+13=0$

35. 0

36. $a^2+b^2=1$

37. $x^2+y^2=1$

38. 2

39. -1

40. 4

41. 2

42. 49

43. Given that ${}^8P_r=56$

$\Rightarrow \dfrac{8!}{(8-r)!}=56$

$\Rightarrow (8-r)!=\dfrac{8!}{56}=6!$

$\Rightarrow 8-r=6$

$\Rightarrow r=8-6=2$

44. The number of ways in which 5 persons can sit in a round table = (5-1)!
= 4! = 4 . 3 . 2 . 1 = 24.

45. Each letter can be posted in any one of the 7 letter boxes.

Required number of ways

= 7 x 7 x 7 x 7 x 7 = 7^5.

46. ${}^nP_6=20.{}^nP_4$

$\Rightarrow$ (n - 4) (n - 5) = 20

$\Rightarrow$ n^2 - 9n + 20 = 20

$\Rightarrow$ n (n - 9) = 0

$\Rightarrow$ n = 9

(n = 0 is rejected)

47. ${}^nC_{14}={}^nC_{16}$

$\Rightarrow n=14+16=30$

$\therefore\ {}^{32}C_n={}^{32}C_{30}={}^{32}C_{32-30}$

$={}^{32}C_2=\dfrac{32.31}{1.2}=496$.

48. Given that $n={}^mC_2=\dfrac{m(m-1)}{2}$

$\therefore\ {}^nC_2=\dfrac{n(n-1)}{2}$

$=\dfrac{1}{2}\dfrac{m(m-1)}{2}\left[\dfrac{m(m-1)}{2}-1\right]$

$=\dfrac{m(m-1)}{4}\left[\dfrac{m(m-1)-2}{2}\right]$

$=\dfrac{1}{8}(m-1)(m^2-m-2)$

$=\dfrac{1}{8}(m+1)m(m-1)(m-2)$

$=3.\dfrac{1}{24}(m+1)\,m\,(m-1)\,(m-2)$

$=3.{}^{m+1}C_4$.

49. $^{47}C_4 + \sum_{i=1}^{5} {}^{52-i}C_3$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= \left({}^{47}C_4 + {}^{47}C_3\right) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= \left({}^{48}C_4 + {}^{48}C_3\right) + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= \left({}^{49}C_4 + {}^{49}C_3\right) + \left({}^{50}C_3 + {}^{51}C_3\right)$

$= \left({}^{50}C_4 + {}^{50}C_3\right) + {}^{51}C_3 = {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$.

50. $\frac{1}{{}^4C_n} = \frac{1}{{}^5C_n} + \frac{1}{{}^6C_n}$

$\Rightarrow \frac{n!(4-n)!}{4!} = \frac{n!(5-n)!}{5!} + \frac{n!(6-n)!}{6!}$

$\Rightarrow 1 = \frac{5-n}{5} + \frac{(6-n)(5-n)}{6.5}$

$\Rightarrow 30 = 6(5-n) + (6-n)(5-n)$

$\Rightarrow n^2 - 17n + 3 = 0$

$\Rightarrow (n-2)(n-15) = 0$

$\Rightarrow n = 2$

($\therefore$ n = 15 is rejected).

51. Let (r+1)th term be the constant term.

(r+1)th term $= {}^{15}C_r\,(x^3)^{15-r}\left(-\frac{1}{x^2}\right)^r$

$= (-1)^r\,{}^{15}C_r\;x^{45-5r}$

Since it is independent of x,

we have 45 - 5r = 0

$\Rightarrow r = 9$

Constant term = $(-1)^9\,{}^{15}C_9 = -{}^{15}C_9$.

52. 11

53. 64

54. x = 36

Reason $1 + 6 + 11 + 6 + ... + x = 148$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 148$

$\Rightarrow \frac{n}{2}[2.1 + (n-1)5] = 148$

$\Rightarrow n(5n-3) = 296$

$\Rightarrow 5n^2 - 3n - 296 = 0$

$\Rightarrow (n-8)(5n+37) = 0$

$\Rightarrow n = 8 \quad (\therefore n \neq -\frac{37}{5})$

$\therefore x = a + (n-1)d = 1 + (8-1)5 = 1 + 35 = 36$.

55. $\frac{T_m}{T_n} = \frac{2m-1}{2n-1}$

Reason : Let a be first term and d be the common difference.

$\therefore \quad S_m = \frac{m}{2}[2a + (m-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$\Rightarrow \quad \frac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-a)d]} = \frac{m^2}{n^2}$

$\Rightarrow \quad \frac{2a + (m-1)d}{21 + (n-1)d} = \frac{m}{n}$

$\Rightarrow \quad 2an + (m-1)nd = 2am + (n-1)md$

$\Rightarrow \quad 2a(n-m) = (n-m)d$

$\Rightarrow \quad d = 2a$

$\therefore \quad \frac{T_m}{T_n} = \frac{a_t(m-1)d}{a_t(n-1)d} = \frac{a + (m-1)2a}{a + (n-1)2a} = \frac{2m-1}{2n-1}$

56. 0

57. 4^5

Reason

Let a be the first term and 4 be the common ratio.

The third term = 4

$\Rightarrow ar^2 = 4$

The first five terms are a, ar, ar^2, ar^3, ar^4.

Prodcut of first five terms

$= a\,.\,ar - ar^2.\ ar^3.\ ar^4$

$= a^5 r^{10} = (ar^2)^5 = 4^5$.

58. 1

59. $\frac{a+b}{ab}$

Reason

$$A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$A_2 = a + \frac{2(b-a)}{3} = \frac{2b+a}{3}$$

$$G_1 = a\,.\left(\frac{b}{a}\right)^{\frac{1}{3}} = a^{\frac{2}{3}}\,.\,b^{\frac{1}{3}}$$

$$G_2 = a\,.\left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{3}}\,.\,b^{\frac{2}{3}}$$

$$\therefore \frac{A_1 + A_2}{G_1 G_2} = \frac{\frac{3(a+b)}{3}}{ab} = \frac{a+b}{ab}$$

60. 2e

61. 2

62. 60

63. $\pm 4\sqrt{2}$

64. 8

65. $\frac{3}{5}$

66. $x = 0$

67. 5

68. (-6, 3, -4)

69. $\sqrt{b^2 + c^2}$

70. 3 : 2 externally

71. -1

Reason

$$= \lim_{\theta \to \frac{\pi}{2}} \frac{\cos\theta}{\theta - \frac{\pi}{2}}$$

$$= \lim_{x \to 0} \frac{\cos(\frac{\pi}{2} + x)}{x}$$

$$= \lim_{x \to 0} -\frac{\sin x}{x} \qquad [\theta - \frac{\pi}{2} = x]$$

$$= -\lim_{x \to 0} \left(\frac{\sin x}{x}\right) = -1.$$

72. $\frac{1}{3}$

73. 100

Reason

$$\lim_{x \to \infty} \frac{(x+1)^{100} + (x+2)^{100} + ... + (x+100)^{100}}{x^{10} + 10^0}$$

$$= \lim_{x \to \infty} \frac{\left(1 + \frac{1}{x}\right)^{10} + \left(1 + \frac{2}{x}\right)^{10} + ... + \left(1 + \frac{100}{x}\right)^{10}}{1 + \left(\frac{10}{x}\right)^{10}}$$

[Dividing x^{10} both in the numerator and denomination.]

= 100

74. log 5. log 2

Reason

$$\lim_{x\to 0}\frac{10^x-2^x-5^x+1}{x\tan x}$$

$$=\lim_{x\to 0}\frac{2^x(5^x-1)-1(5^x-1)}{x\tan x}$$

$$=\lim_{x\to 0}\frac{(5^x-1)(2^x-1)}{x^2-\left(\frac{\tan x}{x}\right)}$$

$$=\lim_{x\to 0}\frac{5^x-1}{x}.\lim_{x\to 0}\frac{2^x-1}{x}.\lim_{x\to 0}\frac{1}{\left(\frac{\tan x}{x}\right)}$$

$$=\log 5.\log 2.$$

75. $\frac{1}{8}$

Reason

$$\lim_{x\to 0}\frac{1-\text{Cos}(1-\text{Cos}x)}{x^4}$$

$$=\lim_{x\to 0}\frac{1-\text{Cos}(2\,\text{Sin}\frac{2x}{2})}{x^4}$$

$$=\lim_{x\to 0}\frac{2\,\text{Sin}^2(\text{Sin}^2\frac{2x}{2})}{x^4}$$

$$=2\lim_{x\to 0}\left[\frac{\text{Sin}\,(\text{Sin}^2\frac{x}{2})}{x^2}\right]^2$$

$$=2\lim_{x\to 0}\left[\frac{\text{Sin}\left(\text{Sin}^2\frac{x}{2}\right)}{\text{Sin}^2\frac{x}{2}}.\frac{\left(\text{Sin}^2\frac{x}{2}\right)}{\frac{x^2}{4}}.\frac{1}{4}\right]^2$$

$$=\frac{1}{8}.$$

76. $\frac{\pi}{180}$

77. -1

78. $a\,x^{a-1}$

Reason

$$\frac{d(e^{a\log x}+e^{a\log a})}{dx}$$

$$=d\frac{(e^{\log x^a}+e^{\log a^a})}{dx}$$

$$=\frac{d}{x}(x^a+a^a)=ax^{a-1}.$$

79. -2

80. 0

Reason

$$y=\text{Sec}^{-1}\left(\frac{x+1}{x-1}\right)+\text{Sin}^{-1}\left(\frac{x-1}{x+1}\right)$$

$$\Rightarrow y=\text{Cos}^{-1}\left(\frac{x-1}{x+1}\right)+\text{Sin}^{-1}\left(\frac{x-1}{x+1}\right)$$

$$=\frac{\pi}{2}$$

$$\Rightarrow\frac{dx}{dy}=\frac{d\left(\frac{\pi}{2}\right)}{dx}=0\cdot$$

81. $\sim(p\vee\sim q)=\sim p\wedge q$

82. $\sim q\to p$

83. $\sim q\to p$

84. $x=0\to \text{Sin}\,x=0$

85. 20

Reason

The data is 30, 40, 85, 75 45.

Let $\overline{x}$ be the arithmetic mean.

$$\overline{x} = \frac{30+40+85+75+45}{5} = \frac{275}{5} = 55$$

To find the mean deviation, we form the following table.

x_i	$x_i - \overline{x}$	$\lvert x_i - \overline{x}\rvert$
30	–25	25
40	–15	15
85	30	30
75	20	20
45	–10	10

Mean Deviation $= \sum_{i=1}^{n} \frac{\lvert x_i - \overline{x}\rvert}{5} = \frac{100}{5} = 20$

86. 2.8

Reason

The given data is tabulated as below :

Age x_i	Frequency f_i	$f_i\, x_i$	$\lvert x_i - \overline{x}\rvert$	$f_i \lvert x_i - \overline{x}\rvert$
14	5	70	2	10
15	4	60	1	4
30	1	30	14	14

$$\sum f_i = 10,\ \sum f_i x_i = 160,\ \sum f_i \lvert x_i - \overline{x}\rvert = 28$$

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{160}{10} = 16$$

Mean deviation $= \frac{\sum f_i \lvert x_i - \overline{x}\rvert}{\sum f_i} = \frac{28}{10} = 2.8$

87. If x_1, x_2, ... x_n be the variables with the corresponding frequenices, f_1, f_2, ..., f_n and $\overline{x}$ be their arithmetic mean, then the standard deviation is defined as

$$S.D. = \sqrt{\frac{\sum_{c=1}^{n} f_i (x_i - \overline{x})^2}{\sum_{i=1}^{n} f_i}}$$

88. If $\overline{x}$ be the mean and σ be the standard deviation then the coefficient of variation is defined as

$$C.V. = \frac{\sigma}{x} x100 .$$

89. If A be an event assocaited with a sample sapce S, then S - A defined as the complementary event of A. It is denoted by A^c.

$$\therefore A^c = S - A .$$

90. $\frac{1}{4}$

Reason

If two coins are tossed, then the sample space

$$s = \{hh, ht, th, tt\}$$

$$\therefore \lvert S\rvert = 4$$

Let A be the event of getting two heads.

$$A = \{44\}, \quad \therefore \lvert A\rvert = 1 .$$

$$P(A) = \frac{\lvert A\rvert}{\lvert S\rvert} = \frac{1}{4} .$$

91. $\frac{1}{36}$

Reason

Three dice are rolled.

$\therefore |S| = 6 \times 6 \times 6 = 216$

Let A be the event that the same number will appear in all dice.

$\therefore |A| = 6$

$$P(A) = \frac{|A|}{|S|} = \frac{6}{216} = \frac{1}{36}$$

92. $\frac{1}{12}$

Reason

Let S be the sample space when two cards are drawn.

$|S| = C(52, 2)$

Let A be the event such that both cards are spades.

$\therefore |A| = C(13, 2)$

$$P(A) = \frac{|A|}{|S|} = \frac{C(13,2)}{C(52,2)} = \frac{13.12}{52.51} = \frac{1}{17}.$$

93. $\frac{4}{9}$

Reason

There are 9 one digit numbers.

4 integers are even, i.e. 2, 4, 6, 8

Required Probability $= \frac{4}{9}$.

94. $\frac{1}{18}$

Reason

Two dice are thrown.

$\therefore |S| = 6 \times 6 = 36$

Let A be the event of getting of total of 11.

$A = \{56, 65\} \quad \therefore |A| = 2$

$$P(A) = \frac{|A|}{|S|} = \frac{2}{36} = \frac{1}{18}$$

95. $\frac{1}{8}$

Reason

Acon is tossed three times

$|S| = 8$

Let A be an event of getting all heads.

$A = \{444\} \qquad \therefore |A| = 1$

$P(A) = \frac{1}{8}$.

96. $\frac{1}{17}$

97. $\frac{1}{6}$

98. $\frac{1}{2}$

99. {H, T1, T2, T3, T4, T5, T6}

100. $\frac{1}{2}$

GROUP - B

SHORT TYPE QUESTIONS

Each questions carries 4 Marks.

2. Sets, Relation & Functions, Trigonometric functions.

1. If A and B are two sets then prove that $(A\bigcup B)' = A'\bigcap B'$.
2. If A and B are two sets then prove that $(A\bigcap B)' = A'\bigcup B'$.
3. Prove that $A - B = B' - A'$.
4. Prove that $A - B = A\bigcap B'$.
5. If A, B and C be three sets, then prove $A-(B\bigcup C) = (A-B)\bigcap (A-C)$.
6. If A, B and C be three sets, then prove $A\bigcap (B-C) = (A\bigcap B)-(A\bigcap C)$.
7. If A, B and C be three sets, then prove that $A \text{ x } (B\bigcup C) = (A \text{ x } B)\bigcup (A \text{ x } C)$.
8. 50 students play cricket or football. 28 students play cricket and 32 students play football. How many students play both cricket and football ?
9. In a club, 28 players play football, 34 players play cricket and 12 play both. If all the membres of the club play at least one of the above games, find the numbr of members in the club.
10. Prove that $A\bigcap (B\Delta C) = (A\bigcap B)\Delta (A\bigcap C)$.
11. Prove that $A\Delta B = (A\bigcup B)-(A\bigcap B)$.
12. Prove that $A-(B\bigcup C) = (A-B)-C$.
13. Prove that $(A\bigcap B)-C = A\bigcap (B-C)$.
14. If X and Y are two sets such that $X\bigcup Y$ has 20 objects, X has 10 objects and Y has 15 objects. How many objects does $X\bigcap Y$ have ?
15. In a group of 450 people, 300 can speak Hindi, 250 can speak English. How many people can speak both Hindi and English ?
16. Let $A = \{4,5,6,7\}$, $B = \{2,3,5,8\}$. A relation R from A to B is defined by
    $R = \{x, y) : x \in A,\ \ y \in B \text{ and } x + y \leq 10\}$.
    Describe R in Roster from. Find its domain and range.
17. Let $A = \{5,7,10,11\}$ and $B = \{4,6,8\}$. A relation R from A to B is defined by
    $R = \{(x, y) : x \in A,\ y \in B \text{ and } x^2 + y^2 = 100\}$. Describe R in roster form. Find its domain and range.
18. A relation R is defined from the set $A = \{2,3,4,5\}$ to a set $B = \{3,6,7,10\}$ as
    $R = \{(x, y) : x \in A, y \in B$
    and x is relatively prime to y$\}$.
    Write R as a set of ordre pairs and its domain and range.
19. Let A={1,2,3,4,5,6}.
    Let a relation R on a set A is defined by
    R={a,b}: $a, b \in A$ and b is exactly divsible by a}
    Write R is roster form.
20. Let A = {0, 1, 2, 3, 4, ...,9, 10}
    A relation R on the set A is defined by
    $R = \{(x, y) : x, y \in A \text{ and } 2x + 3y = 12\}$
    Express R is roster form.
21. If A = {-2,-1,0,1,2} and $f : A \to R$ defined by $f(x) = x^2 + 2$, then find the range of f.

22. If A = {-1,0,1,2,3} and $f : A \to R$ defined by f(x) = x^3 + x - 2, find f(A).

23. If $f(x) = x + \frac{1}{x}$, then show that

$[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$.

24. If $f(x) = x^3 - \frac{1}{x^3}$ then show that

$f(x) + f\left(\frac{1}{x}\right) = 0$.

25. If $f(x) = \frac{x+1}{x-1}$ then shwo that $f[f(x)] = x$.

26. If $f(x) = (a - x^n)^{\frac{1}{x^n}}$ when $a > 0$ and n is a positive integer, then show that $f[f(x)] = x$.

27. If Sec A - tan A = p, $(p \neq 0)$, then show that

(i) $\text{Sin } A = \frac{1-p^2}{1+p^2}$

(ii) $\tan A = \frac{1-p^2}{2p}$

(iii) $\text{Sec } A = \frac{p^2+1}{2p}$

28. If a Cos A - b Sin A = c then prove that $a \text{Sin} A + b \text{Cos} A = \pm\sqrt{a^2 + b^2 - c^2}$.

29. If $\text{Cos}\,\theta + \text{Sin}\,\theta = \sqrt{2}\text{Cos}\,\theta$, then prove that $\text{Cos}\,\theta - \text{Sin}\,\theta = \sqrt{2}\text{Sin}\,\theta$.

30. Prove taht $\frac{1 + \text{Sin}\,\theta - \text{Cos}\,\theta}{1 + \text{Sin}\,\theta + \text{Cos}\,\theta} = \tan\frac{\theta}{2}$.

31. If $\tan^2\theta = 2\tan^2\varnothing + 1$, then prove that $\text{Cos}\,2\theta + \text{Sin}^2\varnothing = 0$.

32. Prove that

$\text{Cos}\,20^0\ \text{Cos}\,40^0\ \text{Cos}\,60^0\ \text{Cos}\,80^0 = \frac{1}{16}$.

33. Show that $\tan 7\frac{1}{2}^0 = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$.

34. If $A + B + C = \pi$, then prove that

$\text{Cos}^2 A + \text{Cos}^2 B + \text{Cos}^2 C$
$= 1 - 2\,\text{Cos}\,A\ \text{Cos}\,B\ \text{Cos}\,C$

35. Find the maximum value of $5\,\text{Sin}\,x + 12\,\text{Cos}\,x$.

36. Solve $\text{Cos}^2 x + 3\,\text{Sin}\,x = 3$.

37. Solve $2\,\text{Cos}^2 x + 4\,\text{Sin}^2 x = 3$.

38. In any triangle ABC, prove that

$a(\text{Sin}\,B - \text{Sin}\,C) + b(\text{Sin}\,C - \text{Sin}\,A)$
$+ C(\text{Sin}\,A - \text{Sin}\,B) = 0$

39. If $\text{Cos}\,B = \frac{\text{Sin}\,C}{2\,\text{Sin}\,A}$, then show that the triangle is isosceles.

40. In any triangle ABC, prove that

$b\,\text{Sin}\,B - c\,\text{Sin}\,C = a\,\text{Sin}\,(B - C)$.

41. In any triangle ABC, prove that

$c^2 = (a-b)^2\,\text{Cos}^2\frac{c}{2} + (a+b)^2\,\text{Sin}^2\frac{c}{2}$.

42. In any triangle ABC, prove that

$c(a\,\text{Cos}\,B - b\,\text{Cos}\,A) = a^2 - b^2$.

43. In any triangle ABC, show that

$(b^2 - c^2)\,\text{Cot}\,A + (c^2 - a^2)$
$\text{Cot}\,B + (a^2 - b^2)\,\text{Cot}\,C = 0$

44. In any triangle ABC, prove that

$\frac{b^2 - c^2}{a}\text{Cos}\,A + \frac{c^2 - a^2}{b}\text{Cos}\,B$
$+ \frac{a^2 - b^2}{c}\text{Cos}\,C = 0$

45. In any triangle ABC, prove taht

$\frac{b^2 - c^2}{\text{Cos}\,B + \text{Cos}\,C} + \frac{c^2 - a^2}{\text{Cos}\,C + \text{Cos}\,A}$
$+ \frac{a^2 - b^2}{\text{Cos}\,A + \text{Cos}\,B} = 0$

3. Principle of Mathematical Induction, Complex Numbers and Quadratic Equations, Linear Inequalities, Permutation and Combination, Binomal theorem, Sequence and Series.

Prove by the principle of conduction (No. 1 to 6)

1. $1+2+3+...+n=\frac{(n+1)}{2}$ for $n\in N$.

2. $1+3+5+...+(2n-1)=n^2$ for $n\in N$.

3. $n(n+1)(n+2)$ is divisible by 6 for all $n\in N$.

4. $3^{2n}-1$ is divisible by 8 for all $n\in N$.

5. n^3-n is a multiple of 6 for $n\in N$.

6. n(n+1)(n+5) is divisible by for $n\in N$.

7. Find the square root of 7 - 24i.

8. Find the least positive value of n if $\left(\frac{1+i}{1-i}\right)^n=1$.

9. If $(x+iy)^{\frac{1}{3}}=a+ib,\ a,b,\ x,\ y\in R$ then show that $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$.

10. If $\frac{a+ib}{c+id}=x+iy$, then show that $\frac{a^2+b^2}{c^2+d^2}=x^2+y^2$.

11. Solve the equation $9x^2 - 12x + 20 = 0$ by factorization method.

12. If $1,\ \omega,\ \omega^2$ are three cube roots of unity then show that $(1-\omega+\omega^2)(1+\omega-\omega^2)=4$.

13. Show that $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)=9$.

14. Show that $(1+\omega-2\omega^2)^4+(4+2+4\omega^2)^4=81$.

15. Expenses $\frac{a+ib}{a-ib}-\frac{a-ib}{a+ib}$ in the form A+iB.

16. If Z_1 and Z_2 are complex numbers, then prove that $\overline{Z_1Z_2}=\overline{Z_1}.\ \overline{Z_2}$.

17. Express $(1+a^2)(1+b^2)(1+c^2)$ as the sum of two squares.

18. Solve the quadrate equations $x^2-6x+25=0$.

19. Solve that $\frac{1}{(3+i)^2}+\frac{1}{(3-i)^2}=\frac{4}{25}$.

20. Find the value of $x^4-4x^3+7x^2-6x+3$ if $x=1+i$.

21. Show that $(2+5\omega+2\omega^2)^6=2+2\omega+5\omega^2)^6$=729.

22. Show that $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})=49$

23. Using principle of induction, show that $2^{3n}-1$ is multiple of 7 for all $n\in N$.

24. Show that $(1-i)^n\left(1-\frac{1}{i}\right)^n=2^n$.

25. If $z=x+iy$ then show that the equation $\left|\frac{2z-i}{z+1}\right|=m$ does not represent a circle when m=2.

26. Solve the inequation $3(x-1)\le 2(x-3)$.

27. Solve the inequation $x+\frac{x}{2}+\frac{x}{3}<11$.

28. Solve the inequation $x+12<4x-2$.

29. Find all pairs of consecutive odd positive integers both of which are smaller than 10 and their sum is more than 11.

30. A student in Class XI obtained 70 and 75 marks in first two unit tests. Find the minimum mark he should get in 3rd test to have an average of at least 60 marks.

31. If the set A and B have m and n elements respectively, then how many functions can be defined from A to B ?

32. If $p(n,5)=20p(n,3),\ n>4,$ then find n.

33. If $p(m+n,2)=56$, $p(m-n,\ 2)=12$, then find m and n.

34. Prove that $C(n,r)+C(n,r-1)=C(n+1,r)$.

35. If $C(2n,r)=C(2n,r+2)$, then find r.

36. A genteleman has 15 acquintances in a town of whom 9 are relatives. In how many ways can be invite 8 quests so that 6 of them are relatives ?

37. From among 8 gentlemen and 4 ladies, a committee of five is to be formed. In howmany ways can this be done so as to include at least one lady ?

38. Find the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

39. Find the middle term in the expansion of $\left(2x^2-\frac{1}{x}\right)^7$.

40. Prove that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5...(2n-1)}{n!}.2^n x^n$.

41. Show that

$$C_1+2.C_2+3.C_3+...+nC_n=n2^{n-1}.$$

42. If p^{th}, q^{th} and r^{th} term of on A.P. are a, b, c respectively, then show that

(q - r) a + (r - p) b + (p - q) c = 0.

43. If the 10^{th} term of an A.P. is 52 and 16^{th} term is 82, then find the 30^{th} term.

44. Show that $\sum_{n-1}^{\infty}\frac{n^2}{n!}=2e$.

45. Prove that

$$\frac{1}{2}-\frac{1}{2.2^2}+\frac{1}{3.2^3}-\frac{1}{4.2^4}+...=\log_e\left(\frac{3}{2}\right)$$

4. Straight lines, Conic sections, Introduction to three dimensional geometry.

1. The vertices of a triangle are (0, -4) and (6, 0). It the medians meet at the point (2,0), find the coordinates of the 3^{rd} vertex.

2. Find the ratio in which the line segment joining (-2, -3) and (5, 4) is divided by the x - axis.

3. It the points (x, y) are equidistant from the points (a+b, b-a) and (a-b, a+b), then prove that bx=ay.

4. Find the equation of straight line bisecting the line segment joining (3, -4) and (1, 2) at right angles.

5. Find the equation of the line passing through (-4, 2) and parallel to the line 4x - 3y = 10.

6. It three points (h, 0), (a, b) and (0, k) lies on a line, then show that $\frac{a}{h}+\frac{b}{k}=1$.

7. Find the coordinates of the centroid of the triangle ABC where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the coordinates of A, B and C respectively.

8. Prove that the points (1, 1), (-2, 4), (-1, 2) and (2, -1) taken in order are the vertius of a parallelogram.

9. Prove that the points (7, 2), (1, -2) and (-2, -4) are collinear.

10. Find the coordinates of the point which divide the join of the points (-2, 3) and (5, -7) in the ratio 2 : 3 internally.

11. If P(a, b) is the mid point of a line segment between axes, then show that the equation of the line $\frac{x}{a}+\frac{y}{b}=2$.

12. Find the equation of the line which is perpendicular to the line $2x - 3y + 4 = 0$ and passes through the point of intersection of the lines $2x + y - 3 = 0$ and $x + 2y - 3 = 0$.

13. Find the equation of the line which is parallel to the line $x + y + 1 = 0$ and whose y-intercept is equal to that of $2x - y - 3 = 0$.

14. The perpendicular from the origin to a line meets it at the point (-2, 9). Find the equation of the line.

15. Find the equation of the line whose x-intercept is same as that of the line $x - 2y - 4 = 0$ and y-intercept is same as that of the line $y - 2x - 3 = 0$.

16. Find the equation of the cirlce whose centre is (3, 2) and the circle is tangent to x-axis.

17. Find the equation of the circle whose centre is (-1, 4) and the circle is tangent to y-axis.

18. Found the equation of the circle whose centre is on the line $2x+y-3=0$ and the circle passes through the points (5, 1) and (2, -3).

19. Find the centre and radius of the circle $4x^2 + 4y^2 - 4x + 12y - 15 = 0$.

20. Find the equation of the circle passing through three points (0, 0), (0, 2) and (-1, 0).

21. Find the equation of the parabola whose vertex is (2, 3) and focus is (-2, 3).

22. Find the equation of the parabola passing through the points (1, 2), (-2, 3) and (2, -1) and the axis being parallel to x-axis.

23. Find the equation of the parabola whose focus is (1, 2) and directrix is $x + y = 2$.

24. Find the equation of the parabola whose vertex is (6, -2) and focus is (-3, -2).

25. Find the equation of the parabola whose axis is vertical and parabola passes through the points (0, 2), (-1, 1) and (2, 10).

26. Find the equation of the ellipse whose centre is (0, 0), one vertex is (0, -5) and one end of the minor axis is (3, 0).

27. Find the equation of the ellipse whose foci at $(\pm 5, 0)$ and the length of the major axis is 12.

28. Find the equation of the ellipse whose vertices at $(\pm 5, 0)$ and length of latus rectum is $\frac{8}{5}$.

29. Find the equation of the ellipse whose centre is (5, 4) and the major axis is of length 16 and the minor axis is of length 10.

30. Find the equation of the ellipse in its simpler form if it passes, through the points (-3, 2) and (5,-1),

31. Find the equation of the hyperbola where the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$.

32. Obtain the equation of the hyperbola whose foci at $(\pm 4, 0)$ and verties $(\pm 2, 0)$.

33. Find the equation of the hyperbola whose centre is (0, 0), transverse axis along x-axis of length 4 and focus at $(2\sqrt{5}, 0)$.

34. Find the foci, the vertices, eccentricity and latus rectum of the parabola $9x^2 - 16y^2 = 144$.

35. Find the equation of the hyprebola whose foci at $(\pm 2\sqrt{3}, 0)$ and eccentricity $\sqrt{3}$.

36. Find the perimeter of the triangle whose vertices are (0, 1, 2), (2, 0, 4) and (-4, -2, 7).

37. Show that the points (a, b, c), (b, c, a), (c, a, b) from an equilateral triangle.

38. Show tht the points (2, 3, 2), (5, 5, 6) and (-4, -1, -6) are collinear.

39. Find the lous of the point which are equidistant from the points (1, 2, 3) and (3, 2, -1).

40. Find the ratio in which the line segment through (1, 3, -1) and (2, 6, -2) is divided by Zx-plane.

41. Find the coordinates of the centroid of the triangle with its verties at (a_1, b_1, c_1), (a_2, b_2, c_2) and (a_3, b_3, c_3).

42. Show that the points (0, 7, -10), (1, 6, -6) and (4, 9, -6) are the vertices of an ososceles triangle.

43. Find the coordinates of the point which divides the line joining the points (-2, 3, 5) and (1, -4, 6) in the ratio 2 : 3 externaly.

44. Find the ratio in which the yz - plane divides the line segment formed by joining the points (-2, 4, 7) and (3, -5, 8).

45. Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled triangle.

5. Limits and Derivatives

1. Examine the existence of the following limits.

(i) $\lim_{x \to 2} [x]$

(ii) $\lim_{x \to 4} x - [x]$

(iii) $\lim_{x \to 3} \frac{|x-3|}{x-3}$

(iv) $\lim_{x \to \sqrt{2}} [x] + 3$

Evaluate the following limits.

2. $\lim_{x \to 2} \frac{x^3 - 8}{x^5 - 32}$

3. $\lim_{x \to 0} \frac{(3+x)^3 - 27}{x}$

4. $\lim_{x \to 1} \frac{x^m - 1}{x^n - 1}$, m, n are integers

5. $\lim_{x \to \infty} \frac{n^2 + n + 1}{5n^2 + 2n + 1}$

6. $\lim_{x \to \infty} \frac{1 + 2 + 3 + ... + n}{n^2}$

7. $\lim_{x \to \frac{\pi}{2}} \frac{1 - \text{Sin}\, x}{\left(\frac{\pi}{2} - x\right)^2}$

8. $\lim_{x\to 0} \dfrac{1-\cos^3 x}{x \sin 2x}$

9. $\lim_{x\to 0} \dfrac{1+\sin x-\cos x}{1-\sin x-\cos x}$

10. $\lim_{x\to \frac{\pi}{2}} \left(\dfrac{\pi}{2}-x\right)\tan x$

11. $\lim_{x\to 0} \dfrac{\cos x-\cos 5x}{\cos 2x-\cos 6x}$

12. $\lim_{x\to 0} \dfrac{2\sin x-\sin 2x}{x^3}$

13. $\lim_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$

14. $\lim_{x\to 2} \dfrac{\log_e(x-1)}{x^2-3x+2}$

15. $\lim_{x\to 5} \dfrac{\sqrt{x-1}-2}{x-5}$

16. $\lim_{x\to 0} \dfrac{3^x-2^x}{4^x-3^x}$

17. $\lim_{x\to 1} \dfrac{2^{x-1}-1}{\sqrt{x}-1}$

18. $\lim_{x\to 0} \dfrac{a-\sqrt{a^2-x^2}}{x}$

19. $\lim_{x\to \frac{\pi}{2}} \dfrac{\tan 2x}{x-\frac{\pi}{2}}$

20. $\lim_{x\to 0} \dfrac{\sin ax+bx}{ax+\sin bx}$

21. $\lim_{x\to 0} \dfrac{\sqrt{a^2+x^2}-\sqrt{a^2-x^2}}{x^2}$

22. $\lim_{x\to b} \dfrac{\sqrt{x-a}-\sqrt{b-a}}{x^2-b^2}$

23. Find the derivative of Sec x w.r.t.x from the 1st principle.

24. Find the derivative $\dfrac{1}{x^2}$ w.r.t.x. from the 1st principle.

25. Find the derivative of x Sin x w.r.t.x. from the 1st principle.

26. If $f(x)=|x|+|x-1|$ then find $\dfrac{d\,f(x)}{dx}$.

Differentiate the following functions

27. $y=\dfrac{x+\sin x}{1+\cos x}$

28. $y=x^2\cos x$

29. $y=\dfrac{x^n}{\tan x}$

30. $y=\dfrac{1-\tan x}{1+\tan x}$

31. $y=\dfrac{\cos x}{1+x^2}$

32. $y=\tan^2 x+a^x$

33. $y=\dfrac{e^x+e^{-x}}{x^2+1}$

34. $y=\dfrac{2x+1}{x^2+1}$

35. $y=x^3 \sin x\ e^{4\ln x}$

36. $y = \tan^2 x + \text{Sec}^2 x$

37. $y = \text{Cos}^2 x + e^x \text{Cos}\, x$

38. $y = ax^2 + b\tan x + \ln x^3$

39. $y = \dfrac{\text{Sec}^2 x}{\tan x + \cot x}$

40. $y = \dfrac{\cos 2x}{\tan x - \cot x}$

41. $y = \dfrac{\cos x}{1 + \text{Sin}\, x}$

42. $y = \sqrt{1 + \text{Sin}\, 2x}$

43. $y = \sqrt{1 - \text{Cos}\, 2x}$

44. $y = \sqrt{\dfrac{\sec x + \tan x}{\sec x - \tan x}}$

45. Show that

$$f(x) = \begin{cases} x\,\text{Sin}\dfrac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

is not differentiable at x = 0.

6. Mathematical Reasoning, Statistics and Proability

1. Write the contrapositive and converse of the statement

" $\text{Cos}\, x = 0 \rightarrow x = \dfrac{\pi}{2}$.

2. Write the contrapositive and Converse of the statment

" $2x + 3 = 9 \rightarrow x \neq 5$

3. Find the connectives and component statements of the following compound statements and check whether they ar etrue or false.

   (i) 30 is divisible by 2, 3 and 5.

   (ii) All rational or irrational numbers are real numbers.

   (iii) The constituents of water are oxygen and nitrogen.

4. Show that the statement " For any real number a and b, "$a^2 = b^2$ implies that a = b" is not true by giving a counter example.

5. By giving counter example show that the following statment is not true.

"The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

6. Show the following statement is true by the method of contrapositive.

"If x is an integer and x^2 is odd then x is odd."

7. Write the contrapositive and converse of the following statement.

   (i) x is an even number implies that x is divisible by 4.

   (ii) If a number n is even then n^2 is even.

8. Use truth table to verify

$p \wedge (\sim q \vee q) = p$

Using truth table, verify the following

9. $\sim p \Leftrightarrow q \equiv p \Leftrightarrow \sim q$

10. $\sim (p \Leftrightarrow q) \equiv \sim p \Leftrightarrow q$

11. $p \wedge (p \vee q) = p$

12. If p has a truth T, then what can be said about the truth value of $\sim p \wedge q \Rightarrow p \vee q$.

13. If p has a truth value F, then what can be said about the truth value of $p \vee q \Rightarrow \sim p \wedge q$.

14. Show that the following statements are true by the method of contrapositive. If x is an integer and x^2 is even then x is also even.

15. Show that the statement

    "If x real number such that $x^3 + 4x = 0$ then x=0" by direct method.

16. Define Range. From the list of body temperature of a patient, determine the range of fluctuation.

Time	6 AM	10 AM	2 PM	6 PM	10 PM
Tempeature	99	100	104	102	101

[in Fahrenheit (^{0}F)]

17. Calculate the mean deviation of the marks 30, 40, 85, 75, 45 obtained by a student in 5 different examination.

18. Calculate the mean deviation of the following distrbution

Age in years	14	15	30
No. of persons	5	4	1

19. Define the mean deviation of a
    (i) simple distribution
    (ii) frequency distribution and
    (iii) grouped frequency distribution

20. If σ be the standard deviation, prove that

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - \left(\frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \right)^2 .$$

21. Define the coefficient of variation. Calculate the coefficient of variation of daily wages of five worker which are given below.

    Rs. 15, Rs. 20, Rs. 25, Rs. 30, Rs. 35.

22. Calculate the standard deviation of the first n natural numbers.

23. Find the mean deviation of the values 1,2,3,5.

24. Find the standard deviation of the values 1,2,3,5.

25. Find the standard deviation of the data given in the following table.

x_i	1	2	3	...	n
f_i	1	2	3	...	n

26. Find the standard deviation of the data given in the following table.

x_i :	1	3	5	...	$2n-1$
f_i :	1	1	1	...	1

27. Find the mean deviation of the data

    5, 15, 20, 30, 40.

28. Find the mean and variance of the data given in the following table.

x_i :	2	4	6	...	2n
f_i :	1	1	1	...	1

29. Find the mean deviation of the data

    7, 17, 20, 22, 24

30. Find the variance of the dta given in the following table

x_i :	2	4	6	...	2n
f_i :	1	1	1	...	1

31. If A and B are two mutually exclusive event in a sample space S then prove that

    $P(A \cup B) = P(A) + P(B)$.

32. If A and B are two events in a sample space then prove that

    $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

33. If A and B are any two events and $A \subset B$ then prove that $P(A) \leq P(B)$.

34. If A and B are any two events then prove that $P(A - B) = P(A) - P(A \cap B)$.

35. If A^c be the complement of A then prove that $P(A^C) = 1 - P(A)$.

36. A die is thrown twice. Find the probability that the sum of points obtained is 8.

37. A die is thrown 5 times. Find the probability of getting exactly 3 forms.

38. Prove that for any two events A and B,

    $P(A \cap B) \geq P(A) + P(B) - 1$.

39. A card is selected from 100 cards number 1 to 100. If the card is selected at random, find the probability that the number on the card is divisible by 5.

40. Two cards are drawn from a pack of 52 cards. What is the probability that both are diamonds ?

41. A bag contains 6 white and 7 black balls. If two balls are drawn at random, find the probability that both balls are white.

42. A card is selected from 100 cards numbered 1 to 100. If the card is selected at random, find the probability that the number on the card is divisible both by 2 and 5.

43. 8 persons stand is a line at random. What is the probability that two persons X and Y do not stand together ?

44. A bag contains 5 white and 3 black balls. If a ball is drawn at random, find the probability that it is white.

45. A coin is tossed 3 times. Find the probability of getting at most 2 heads.

GROUP - B

ANSWERS

Each questions carries 4 Marks.

2. Sets, Relation & Functions, Trigonometric functions.

1. Let $x \in (A \cup B)'$

$\Leftrightarrow x \notin A \cup B$

$\Leftrightarrow x \notin A$ and $x \notin B$

$\Leftrightarrow x \in A'$ and $x \in B'$

$\Leftrightarrow x \in A' \cap B'$

$\therefore (A \cup B)' \subset A' \cap B'$ .... (1)

and $A' \cap B' \subset (A \cup B)'$ .... (2)

Form (1) and (2), we have

$(A \cup B)' = A' \cap B'$

2. Let $x \in (A \cap B)'$

$\Leftrightarrow x \notin A \cap B$

$\Leftrightarrow x \notin A$ or $x \notin B$

$\Leftrightarrow x \in A'$ or $x \in B'$

$\Leftrightarrow x \in A' \cup B'$

Thus $(A \cap B)' = A' \cup B'$.

3. Let $x \in A - B$

$\Leftrightarrow x \in A$ and $x \notin B$

$\Leftrightarrow x \notin A'$ and $x \in B'$

$\Leftrightarrow x \in B'$ and $x \notin A'$

$\Leftrightarrow x \in B' - A'$.

$\therefore A - B \subset B' - A'$ .... (1)

and $B' - A' \subset A - B$ .... (2)

From (1) and (2), we get $A - B = B' - A'$.

4. Let $x \in A - B$

$\Leftrightarrow x \in A$ and $x \notin B$

$\Leftrightarrow x \in A$ and $x \in B'$

$\Leftrightarrow x \in A \cap B'$

$\therefore A - B = A \cap B'$

5. Let $x \in A - (B \cup C)$

$\Leftrightarrow x \in A$ and $x \notin B \cup C$

$\Leftrightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Leftrightarrow (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$

$\Leftrightarrow x \in A - B$ and $x \in A - C$

$\Leftrightarrow x \in (A - B) \cap (A - C)$.

$\therefore A - (B \cup C) \subset (A - B) \cap (A - C)$ ... (1)

and $(A - B) \cap (A - C) \subset A - (B \cup C)$ ... (2)

From (1) and (2), we get

$A - (B \cup C) = (A - B) \cap (A - C)$.

6. Let $x \in A \cap (B - C)$

$\Leftrightarrow x \in A$ and $x \in B - C$

$\Leftrightarrow x \in A$ and $(x \in B$ and $x \notin C)$

$\Leftrightarrow (x \in A$ and $x \in B)$ and $(x \in A$ and $x \notin C)$

$\Leftrightarrow x \in A \cap B$ and $x \notin A \cap C$

$\Leftrightarrow x \in (A \cap B) - (A \cap C)$

$\therefore A \cap (B - C) \subset (A \cap B) - (A \cap C)$ ... (1)

and $(A \cap B) - (A \cap C) \subset A \cap (B - C)$ ... (2)

From (1) and (2) we have

$A \cap (B - C) = (A \cap B) - (A \cap C)$.

7. Let $(x, y) \in A \times (B - C)$

$\Leftrightarrow x \in A$ and $y \in B - C$

$\Leftrightarrow x \in A$ and $(y \in B$ and $y \notin C)$

$\Leftrightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \notin C)$

$\Leftrightarrow (x, y) \in (A \times B) - (A \times C)$

$\therefore\ A \times (B - C) \subset (A \times B) - (A \times C) \quad ...(1)$

and $(A \times B) - (A \times C) \subset A \times (B - C) \quad ...(2)$

From (1) and (2), we get

$A \times (B - C) = (A \times B) - (A \times C)$

8. Let C and F be the set of students who play cricket and football respectively. According to the question,

$|C \cup F| = 50$

$|C| = 28$

$|F| = 32$

We know $|C \cup F| = |C| + |F| = |C \cap F|$

$\Rightarrow 50 = 28 + 32 - |C \cap F|$

$\Rightarrow |C \cap F| = 60 - 50 = 10$

$\therefore$ 10 students play both cricket and foot ball.

9. Same as No. 8 (Exercise for the students)

10. Let $x \in A \cap (B \Delta C)$

$\Leftrightarrow x \in A$ and $x \in B \Delta C$

$\Leftrightarrow x \in A$ and $(x \in B$ and $x \notin C$ or $x \in C$ and $x \notin B)$

$\Leftrightarrow (x \in A$ and $x \in B$ and $x \notin C)$
or $(x \in A$ and $x \in C$ and $x \notin B)$

$\Leftrightarrow (x \in A \cap B$ and $x \notin A \cap C)$
or $(x \in A \cap C$ and $x \notin A \cap B)$

$\Leftrightarrow x \in (A \cap B) - (A \cap C)$
or $x \in (A \cap C) - (A \cap B)$

$\Leftrightarrow x \in (A \cap B) \Delta (A \cap C)$

Thus $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

11. Let $x \in (A \cup B) - (A \cap B)$

$\Leftrightarrow x \in A \cup B$ and $x \notin A \cap B$

$\Leftrightarrow x \notin A \cup B$ and $(x \notin A$ or $x \notin B)$

$\Leftrightarrow (x \in A$ or $x \in B)$ and $(x \notin A$ or $x \notin B)$

$\Leftrightarrow [x \in A$ and $(x \notin A$ or $x \notin B)]$
or $[x \in B$ and $(x \notin A$ or $x \notin B)$

$\Leftrightarrow (x \in A$ and $x \notin B)$ or $(x \in B$ and $x \notin A)$

$\Leftrightarrow x \in A - B$ or $x \in B - A$

$\Leftrightarrow x \in (A - B) \cup (B - A)$

$\Leftrightarrow x \in A\ \Delta\ B$

$\therefore\ A \Delta B = (A \cup B) - (A \cap B)$

12. Let $x \in A - (B \cup C)$

$\Leftrightarrow x \in A$ and $x \notin B \cup C$

$\Leftrightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Leftrightarrow (x \in A$ and $x \notin B)$ and $x \notin C$

$\Leftrightarrow x \in A - B$ and $x \notin C$

$\Leftrightarrow x \in (A - B) - C$

$\therefore\ A - (B \cup C) = (A - B) - C$.

13. Let $x \in (A \cap B) - C$

$\Rightarrow x \in A \cap B$ and $x \notin C$

$\Rightarrow (x \in A$ and $x \in B)$ and $x \notin C$

$\Rightarrow x \in A$ and $(x \in B$ and $x \notin C)$

$\Rightarrow x \in A$ and $x \in B - C$

$\Rightarrow x \in A \cap (B - C)$

$\therefore\ (A \cap B) - C = A \cap (B - C)$

14. Given that $|X| = 10$, $|Y| = 15$ and $|X \cup Y| = 20$

We know $|X \cup Y| = |X| + |Y| - |X \cap Y|$

$\Rightarrow 20 = 10 + 15 - |X \cap Y|$

$\Rightarrow |X \cap Y| = 25 - 20 = 5$

15. Let H and E be the set of people speaking Hindi and Enlgish respectively. According to the question,

$|H| = 300$

$|E| = 250$

$|H \cup E| = 450$

$\therefore |H \cup E| = |H| + |E| - |H \cap E|$

$\Rightarrow 450 = 300 + 250 - |H \cap E|$

$\Rightarrow |H \cap E| = 550 - 450 = 100$

$\therefore$ The number of people speaking both Hindi and English is 100.

16. Given than A={4, 5, 6, 7}, B={2, 3, 5, 8}

$R = \{(x, y) : x \in A,\ y \in B \text{ and } x+y \le 10\}$

$= \{(4,2),(4,3),(4,5),(5,2),(5,3),(5,5),$
$(6,2),(6,3),(7,2),(7,3)\}$

Domain = {4, 5, 6, 7}

Range = {2, 3, 5}

17. Given that A = {5, 7, 10, 11}, B = {4, 6, 8}

$R = \{(x, y) : x \in A, y \in B \text{ and } x^2 + y^2 \le 100\}$

$= \{(5,4), (5,6), (5,8), (7,4), (7,6)\}$

Domain = {5, 7}

Range = {4, 6, 8}

18. Given that A={2, 3, 4, 5}, B={3, 6, 7, 10}

$R = \{(x, y) : x \in A\ \ y \in B$ and relatively prime to y}

= {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4,7), (5,3), (5,6), (5, 7)}

Domain = {2, 3, 4, 5}

Range = {3, 6, 7, 10}

19. Given that A = {1, 2, 3, 4, 5, 6}

$R = \{(a, b) : a, b \in A$ and b is exactly divisible by a }

= {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (5,5), (6,6)}.

20. Here A = {0, 1, 2, 3, 4, ... 9, 10}

$R = \{(x, y) : x, y \in A \text{ and } 2x + 3y = 12\}$

= {(0,4), (3,2), (6,0)}

21. Given that A={-2, -1, 0, 1, 2}

$f(x) = x^2 + 2$

$\therefore f(-2) = (-2)^2 + 2 = 4 + 2 = 6$

$f(-1) = (-1)^2 + 2 = 1 + 2 = 3$

$f(0) = 0^2 + 2 = 2$

$f(1) = 1^2 + 2 = 1 + 2 = 3$

$f(2) = 2^2 + 2 = 4 + 2 = 6$

$\therefore$ Range of f = {2, 3, 6}

22. Given that A = {-1, 0, 1, 2, 3}

$f(x) = x^2 + x - 2$

$f(-1) = (-1)^2 + (-1) - 2 = 1 - 1 - 2 = -2$

$f(0) = 0^2 + 0 - 2 = -2$

$f(1) = 1^2 + 1 - 2 = 0$

$f(2) = 2^2 + 1 - 2 = 4$

$f(3) = 3^2 + 3 - 2 = 10$

$\therefore f(A) = \{-2, 0, 4, 10\}$.

23. Given that $f(x) = x + \frac{1}{x}$

$\therefore f(x^3) = x^3 + \frac{1}{x^3}$

$[f(x)]^3 = \left(x + \frac{1}{x}\right)^3$

$= x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$= f(x^3) + 3f(X)$ ... (1)

Also $f\left(\frac{1}{x}\right) = \frac{1}{x} + \frac{1}{1/x} = \frac{1}{x} + x = x + \frac{1}{x} = f(x)$.(2)

From (1) and (2), we have

$[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$.

24. Given that $f() = x^3 - \frac{1}{x^3}$ ... (1)

Again $f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{(1/x)^3} = \frac{1}{x^3} - x^3$ ... (2)

Adding (1) and (2), we get

$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0.$

25. Given $f(x) = \frac{x+1}{x-1}$

$\therefore \quad f[f(x)] = \frac{f(x)+1}{f(x)-1} = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$

$= \frac{x+1+x-1}{x+1-(x-1)} = \frac{2x}{2} = x.$

26. Given that $f(x) = (a - x^n)^{\frac{1}{n}}$

$f[f(x)] = [a - [f(x)^n]^{\frac{1}{n}}$

$= [a - \{(a - x^n)^{\frac{1}{n}}\}^n]^{\frac{1}{n}}$

$= [a - (a - x^n)]^{\frac{1}{n}}$

$= (x^n)^{\frac{1}{n}} = x \cdot$

27. Given that $\text{Sec } A - \tan A = p$ ... (1)

We know $\text{Sec}^2 A - \tan^2 A = 1$

$\Rightarrow (\text{Sec } A - \tan A)(\text{Sec } A + \tan A) = 1$

$\Rightarrow p(\text{Sec } A + \tan A) = 1$

$\Rightarrow \text{Sec } A + \tan A = \frac{1}{p}$ ... (2)

Adding (1) and (2), we get

$2 \text{Sec } A = p + \frac{1}{p} = \frac{p^2+1}{p}$

$\Rightarrow \text{Sec } A = \frac{p^2+1}{2p}$

Subtracting (1) from (2), we get

$2 \tan A = \frac{1}{p} - p = \frac{1-p^2}{p}$

$\Rightarrow \tan A = \frac{1-p^2}{2p}$

Again $\frac{\text{Sec } A}{\tan A} = \frac{\frac{p^2+1}{2p}}{\frac{1-p^2}{2p}}$

$\Rightarrow \frac{1}{\text{Sin } A} = \frac{1+p^2}{1-p^2}$

$\Rightarrow \text{Sin } A = \frac{1-p^2}{1+p^2}$

28. Given that a Cos A - b Sin A = C squaring both sides, we get $(a \text{ Cos } A - b \text{ Sin } A)^2 = C^2$

$\Rightarrow a^2\text{Cos}^2 A + b^2 \text{Sin}^2 A - 2ab \text{ Cos } A = C^2$

$\Rightarrow a^2(1 - \text{Sin}^2 A) + b^2(1 - \text{Cos}^2 A)$

$- 2ab \text{ Cos } A \text{ Sin } A = C^2$

$\Rightarrow a^2 - a^2 \text{Sin}^2 A + b^2 - b^2 \text{Cos}^2 A$

$- 2ab \text{ Cos } A \text{ Sin } A = C^2$

$\Rightarrow a^2 \text{Sin}^2 A + b^2 \text{Cos}^2 A$

$+ 2ab \text{ Cos } A \text{Sin} A = a^2 + b^2 - c^2$

$\Rightarrow (a \text{ Sin } A + b \text{ Cos } A)^2 = a^2 + b^2 - c^2$

$\Rightarrow a \text{ Sin } A + b \text{ Cos } A = \pm\sqrt{a^2 + b^2 - c^2} \cdot$

29. Given that $\text{Cos}\,\theta + \text{Sin}\,\theta = \sqrt{2}\,\text{Cos}\theta$

$\Rightarrow \text{Sin}\,\theta = \sqrt{2}\,\text{Cos}\,\theta - \text{Cos}\,\theta = (\sqrt{2} - 1)\text{Cos}\,\theta$

Multiplying both sides by $(\sqrt{2} - 1)$, we get

$(\sqrt{2} + 1)\text{Sin}\,\theta = (\sqrt{2} + 1)(\sqrt{2} - 1)\text{Cos}\,\theta$

$\Rightarrow \sqrt{2}\,\text{Sin}\,\theta + \text{Sin}\,\theta = \text{Cos}\,\theta$

$\Rightarrow \text{Cos}\,\theta - \text{Sin}\,\theta = \sqrt{2}\,\text{Sin}\,\theta \cdot$

30. L.H.S.

$$= \frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \frac{(1-\cos\theta)+\sin\theta}{(1+\cos\theta)+\sin\theta}$$

$$= \frac{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}$$

$$= \tan\frac{\theta}{2}$$

31. Given that $\tan^2\theta = 2\tan^2\varnothing+1\cdot$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-(2\tan^2\varphi+1)}{1+(2\tan^2\varphi+1)}$$

$$= \frac{-2\tan^2\varphi}{2+2\tan^2\varphi} = \frac{-2\tan^2\varphi}{2(1+\tan^2\varphi)}$$

$$= \frac{-\tan^2\varphi}{\sec^2\varphi} = -\frac{\sin^2\varphi}{\cos^2\varphi}\,.\,\cos^2\varphi$$

$$= -\sin^2\varphi$$

$$\Rightarrow \cos 2\theta + \sin^2\varphi = 0$$

32. L.H.S. $= \cos 20^0\ \cos 40^0\ \cos 60^0\ \cos 80^0$

$$= \cos 20^0\ .\ \cos 40^0\,.\,\frac{1}{2}.\ \cos 80^0$$

$$= \frac{1}{4}\ 2\cos 40^0\ .\ \cos 20^0\ .\ \cos 80^0$$

$$= \frac{1}{4}\ [\cos 60^0\ +\ \cos 20^0]\ \cos 80^0$$

$$= \frac{1}{4}\ [\frac{1}{2}\ +\ \cos 20^0]\ \cos 80^0$$

$$= \frac{1}{8}\ \cos 80^0 + \frac{1}{8}\ .\ 2\cos 80^0\ .\ \cos 20^0$$

$$= \frac{1}{8}\ \cos 80^0 + \frac{1}{8}\ [\cos 100^0 + \cos 60^0]$$

$$= \frac{1}{8}\ \cos 80^0 + \frac{1}{8}\ [\cos(180^0-80^0)+\frac{1}{2}]$$

$$= \frac{1}{8}\ \cos 80^0 + \frac{1}{8}\ \cos 100^0 + \frac{1}{16}$$

$$= \frac{1}{8}\ \cos 80^0 + \frac{1}{8}\ \cos(180^0-80^0) + \frac{1}{16}$$

$$= \frac{1}{8}\ \cos 80^0 - \frac{1}{8}\ \cos 80^0 + \frac{1}{16}$$

$$= \frac{1}{16}\,.$$

33. $$\tan 7\frac{1}{2}^0 = \tan\frac{15^0}{2} = \frac{\sin\frac{15^0}{2}}{\cos\frac{15^0}{2}}$$

$$= \frac{2\sin^2\frac{15^0}{2}}{2\sin\frac{15^0}{2}\cos\frac{15^0}{2}} = \frac{1-\cos 15^0}{\sin 15^0}$$

$$= \frac{(2\sqrt{2}-\sqrt{3}-1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2\,\cdot$$

34. Given that $A+B+C=\pi$

L.H.S. $= \cos^2 A + \cos^2 B + \cos^2 C$

$$= \frac{1}{2}(2\cos^2 A + 2\cos^2 B) + \cos^2 C$$

$$= \frac{1}{2}[1+\cos 2A + 1 + \cos 2B] + \cos^2 C$$

$= \frac{1}{2}[2 + \text{Cos}2A + \text{Cos}2B] + \text{Cos}^2C$

$= 1 + \frac{1}{2}.2\,\text{Cos}(A+B)\,\text{Cos}\,(A-B) + \text{Cos}^2C$

$= 1 + \text{Cos}\,(\pi - C)\,\text{Cos}\,(A-B) + \text{Cos}^2C$

$= 1 + (-\text{Cos}\,C)\,\text{Cos}(A-B) + \text{Cos}^2C$

$= 1 - \text{Cos}\,C[\text{Cos}(A-B) - \text{Cos}\,C]$

$= 1 - \text{Cos}\,C[\text{Cos}(A-B) + \text{Cos}\,(A+B)]$

$= 1 - \text{Cos}\,C.2\,\text{Cos}\,A.\text{Cos}\,B$

$= 1 - 2\,\text{Cos}\,A\,\text{Cos}\,B\,\text{Cos}\,C$.

35. Let $5 = r\,\text{Cos}\,\alpha,\ 12 = r\,\text{Sin}\,\alpha$

$\Rightarrow r^2 = 5^2 + 12^2 = 25 + 144 = 169$

$\Rightarrow r = 13$

Given function $= 5\,\text{Sin}\,x + 12\,\text{Cos}\,x$

$= 4\,\text{Cos}\,\alpha\ \text{Sin}\,x + r\,\text{Sin}\,\alpha\ \text{Cos}\,x$

$= r\,\text{Sin}\,(x+\alpha)$

$= 13\,\text{Sin}\,(x+\alpha)$

Maximum value of $\text{Sin}\,(x+\alpha)$ is 1.

Maximum value of $13\,\text{Sin}\,(x+\alpha)$ is 13

$\Rightarrow$ Maximum value of $5\,\text{Sin}\,x + 12\,\text{Cos}\,x$ is 13.

36. $\text{Cos}^2x + 3\,\text{Sin}\,x = 3$

$\Rightarrow 1 - \text{Sin}^2x + 3\,\text{Sin}\,x = 3$

$\Rightarrow \text{Sin}^2x - 3\,\text{Sin}\,x + 2 = 0$

$\Rightarrow (\text{Sin}x - 2)(\text{Sin}\,x - 1) = 0$

Either $\text{Sin}\,x = 2$ or $\text{Sin}\,x = 1$

But $\text{Sin}\,x = 2$ has no solution

$\therefore\ \text{Sin}\,x = 1 = \text{Sin}\,\frac{\pi}{2}$

$\Rightarrow\ x = n\pi + (-1)^n\,\frac{\pi}{2}$.

37. $2\,\text{Cos}^2x + 4\,\text{Sin}^2x = 3$

$\Rightarrow 2\,(1 - \text{Sin}^2x) + 4\,\text{Sin}^2x = 3$

$\Rightarrow 2 - 2\,\text{Sin}^2x + 4\,\text{Sin}^2x = 3$

$\Rightarrow 2\,\text{Sin}^2x = 1$

$\Rightarrow \text{Sin}^2x = \frac{1}{2}$

$\Rightarrow \text{Sin}\,x = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow x = n\pi + \frac{\pi}{4} = (4n+1)\frac{\pi}{4}, n \in Z$

38. We know $\frac{a}{\text{Sin}\,A} = \frac{b}{\text{Sin}\,B} = \frac{c}{\text{Sin}\,C} = 2R$

$\therefore\ a = 2R\,\text{Sin}\,A,\ b = 2R\,\text{Sin}\,B,\ c = 2R\,\text{Sin}\,C$

LHS $= a(\text{Sin}\,B - \text{Sin}\,C) + b(\text{Sin}\,C - \text{Sin}\,A)$

$+ c(\text{Sin}\,A + \text{Sin}\,B)$

$= 2R\,\text{Sin}\,A(\text{Sin}\,B - \text{Sin}\,C)$

$+ 2R\,\text{Sin}\,B(\text{Sin}\,C - \text{Sin}\,A)$

$+2R\,\text{Sin}\,C(\text{Sin}\,A - \text{Sin}\,B)$

$= 2R[\text{Sin}\,A.\text{Sin}\,B - \text{Sin}A\text{Sin}C$

$+ \text{Sin}\,B.\text{Sin}\,C - \text{Sin}\,A.\text{Sin}\,B$

$+ \text{Sin}\,A\text{Sin}\,C - \text{Sin}\,B\,\text{Sin}\,C]$

$= 2R \text{ x } 0 = 0$

39. Given $\text{Cos}\,B = \frac{\text{Sin}\,C}{2\,\text{Sin}\,A}$

$\Rightarrow \frac{c^2 + a^2 - b^2}{2ca} = \frac{c}{2a}$

$\Rightarrow c^2 + a^2 - b^2 = c^2$

$\Rightarrow a^2 - b^2 = 0$

$\Rightarrow a = b$

$\Rightarrow$ The triangle is isosceles.

40. An exercise to the students.
41. An exercise to the students.
42. An exercise to the students.
43. $(b^2 - c^2)\,\text{Cot}\,A$

$= (4R^2\text{Sin}^2B - 4R^2\text{Sin}^2C)\dfrac{\text{Cos}\,A}{\text{Sin}\,A}$

$= 4R^2(\text{Sin}^2B - \text{Sin}^2C)\,.\,\dfrac{\text{Cos}\,A}{\text{Sin}\,A}$

$= 4R^2\,.\,\text{Sin}\,(B+C)\text{Sin}(B-C)\,.\,\dfrac{\text{Cos}\,A}{\text{Sin}\,A}$

$= 4R^2\,\text{Sin}\,(180 - A).\text{Sin}\,(B-C)\,.\,\dfrac{\text{Cos}\,A}{\text{Sin}\,A}$

$= 4R^2\,\text{Sin}\,A.\text{Sin}\,(B-C).\dfrac{\text{Cos}\,A}{\text{Sin}\,A}$

$= 4R^2\,\text{Cos}\,A\,\text{Sin}\,(B-C)\,.$

Similarly $(c^2 - a^2)\,\text{Cot}\,B$

$= 4R^2\,\text{Cos}\,B\,\text{Sin}\,(B-C)$

$(a^2 - b^2)\,\text{Cot}\,C = 4R^2\,\text{Cos}\,C\,\text{Sin}\,(A-B)$

L.H.S. $= 4R^2[\text{Cos}\,A\,\text{Sin}(B-C)$

$+\text{Cos}\,B\,\text{Sin}\,(B-C) + \text{Cos}\,C\,\text{Sin}(A-B)]$

$= 4R^2 = 0 = 0$

44. $\dfrac{b^2 - c^2}{a}\text{Cos}\,A = \dfrac{4R^2\text{Sin}^2B - 4R^2\text{Sin}^2C}{2R\text{Sin}\,A}.\text{Cos}A$

$= 4R^2\dfrac{(\text{Sin}^2\,B - \text{Sin}\,C)}{2R\,\text{Sin}\,A}\,.\,\text{Cos}\,A$

$= 4R^2\dfrac{[\text{Sin}\,(B+C)\,.\,\text{Sin}\,(B-C)]}{\text{Sin}\,A}\,.\,\text{Cos}\,A$

$= 2R\dfrac{\text{Sin}\,A.\text{Sin}\,(B-C)}{\text{Sin}\,A}.\text{Cos}\,A$

$= 2R\,\text{Cos}\,A\,\text{Sin}\,(B-C)$

Similarly $\dfrac{c^2 - a^2}{b}\text{Cos}\,B = 2R\,\text{Cos}\,B\,\text{Sin}(C-A)$

$\dfrac{a^2 - b^2}{c}\text{Cos}\,C = 2R\,\text{Cos}\,C\,\text{Sin}(A-B)$

L.H.S.

$= \dfrac{b^2 - c^2}{a}\text{Cos}\,A + \dfrac{c^2 - a^2}{b}\text{Cos}\,B + \dfrac{a^2 - b^2}{c}\text{Cos}\,C$

$= 2R[\text{Cos}\,A\,\text{Sin}\,(B-C) + \text{Cos}\,B\,\text{Sin}(C-A)$

$+\text{Cos}\,C\,\text{Sin}(A-B)]$

$= 2R \text{ x } 0 = 0$

45. $\dfrac{b^2 - c^2}{\text{Cos}\,B + \text{Cos}\,C} = \dfrac{4R^2\,\text{Sin}^2B - 4R^2\text{Sin}^2C}{\text{Cos}\,B + \text{Cos}\,C}$

$= 4R^2\dfrac{(\text{Sin}^2B - \text{Sin}^2C)}{\text{Cos}\,B + \text{Cos}\,C}$

$= 4R^2\dfrac{[(1 - \text{Cos}^2B) - (1 - \text{Cos}^2C)]}{\text{Cos}\,B + \text{Cos}\,C}$

$= 4R^2\dfrac{(\text{Cos}^2C) - \text{Cos}^2B)}{\text{Cos}\,B + \text{Cos}\,C}$

$= 4R^2\dfrac{(\text{Cos}\,C + \text{Cos}\,B)(\text{Cos}\,C - \text{Cos}\,B)}{\text{Cos}\,B + \text{Cos}\,C}$

$= 4R^2(\text{Cos}\,C - \text{Cos}\,B)$

Similalry

$\dfrac{c^2 - a^2}{\text{Cos}\,C + \text{Cos}\,A} = 4R^2(\text{Cos}\,A - \text{Cos}\,C)$

and $\dfrac{a^2 - b^2}{\text{Cos}\,A + \text{Cos}\,B} = 4R^2(\text{Cos}\,B - \text{Cos}\,A)$

L.H.S.

$= 4R^2(\text{Cos}\,C - \text{Cos}\,B) + 4R^2(\text{Cos}\,B - \text{Cos}\,A)$

$= 4R^2[\text{Cos}\,C - \text{Cos}\,B + \text{Cos}\,A - \text{Cos}\,C$

$+\text{Cos}\,B - \text{Cos}\,A]$

$= 4R^2 \text{ x } 0 = 0$

3. Principle of Mathematical Induction, Complex Numbers and Quadratic Equations, Linear Inequalities, Permutation and Combination, Binomal theorem, Sequence and Series.

Prove by the principle of conduction (No. 1 to 6)

1. Let P(n) be the statement

   " $1+2+3+...+n=\dfrac{n(n+1)}{2}$

   First we shall show that PCD is true.

   L.H.S. of P(1) = 1

   R.H.S. of $P(1)=\dfrac{1(1+1)}{2}=\dfrac{2}{2}=1$

   L.H.S. of P(1)=R.H.S. of P(1)

   So P(1) is true.

   Let P(k) be true.

   i.e. $1+2+3+...+K=\dfrac{k(k+1)}{2}$

   We shall have to show that P(k+1) is true.

   L.H.S. of P(k+1) = 1 + 2 + 3 + ... + k + (k+1)

   $=\dfrac{k(k+1)}{2}+(k+1)$

   $=\dfrac{k(k+1)+2(k+1)}{2}$

   $=\dfrac{(k+1)+2(k+1)}{2}$ = R.H.S. of P(k + 1)

   So P(k+1) is true

   Here we set that

   (i) P(1) is true

   (ii) P(k) is true $\Rightarrow$ P(k + 1) is true.

   So according to the principle of induction, P(n) is true for all $n \in N$.

   i.e. $1+2+3+...+n=\dfrac{n(n+1)}{2}$

2. Let P(n) be the statement

   $1 + 3 + 5 + ... + (2n-1) = n^2$

   First we shall show that P(1) is true.

   L.H.S. of P(1) =1

   R.H.S. of P(1) = 1^2=1

   So P(1) is true

   Let P(k) be true

   i.e. $1 + 3 + 5 + ... + (2k-1) = k^2$ for $k \in N$.

   We shall have to show that P(k+1) is true.

   L.H.S. of P(k+1) = 1+3+5+...+(2k-1)+(2k+1)

   $=k^2+2k+1$

   $=(k+1)^2$ = R.H.S. of P(k+1)

   So P(k+1) is true.

   Therefore P(n) is true for all $n \in N$

   i.e. $1+3+5+...+(2n - 1) = n^2$ for $n \in N$

3. Let P(n) be the statement

   n(n+1)(n+2) is divisible by 6 for all $n \in N$

   First we shall show that P(1) is ture.

   Taking n =1, we get

   1(1+1)(1+2)=1.2.3 = 6 which is divisible by 6.

   So P(1) is ture

   Let P(k) be true

   i.e. k(k+1)(k+2) is divisible by 6

   $\Rightarrow$ k(k+1)(k+2)=6m where $n \in N$

   We shall have show tha P(k+1) is true.

   $\therefore (k+1)(k+1+1)(k+1+2)$

$= (k+1)(k+2)(k+3)$

$= k(k+1)(k+2) + 3(k+1)(k+2)$

$= 6m + 6.\frac{(k+1)(k+2)}{2}$

$= 6\left[m + \frac{(k+1)(k+2)}{2}\right]$

$= 6p \text{ where } p = m + \frac{(k+1)(k+2)}{2} \in N$

which is divisible by 6.
So p(k+1) is true.

Therefore P(n) is true for all $n \in N$ i.e. n(n+1)(n+2) is divisible by 6 for all $n \in N$ Question 4, 5, 6 are exercise for the students.

7. Let $\sqrt{7-24i} = x + iy$

$\Rightarrow 7 - 24i = (x+iy)^2 = x^2 - y^2 + 2ixy$

Equaling the real and imaginary parts, we get

$x^2 - y^2 = 7$ ... (1)

$2xy = -24$ ... (2)

$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$= 7^2 + (-24)^2$

$= 49 + 576$

$= 625$

$\therefore x^2 + y^2 = 25$ ... (3)

(-ve sign is rejected if $x^2 + y^2$ is +ve)

Adding (1) & (3), we get

$3x^2 = 32$

$\Rightarrow x^2 = 16 \quad \Rightarrow x = \pm 4$

Sub tracing (1) from (3), we get

$2y^2 = 16$

$\Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

From (2), we see that if x=4, then y = -2 and if x=-4, then y = 2.

Required square roots are 4-2i and -4+2i.

8. $\left(\frac{1+i}{1-i}\right)^n = 1$

$\Rightarrow \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^n = 1$

$\Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^n = 1$

$\Rightarrow \left(\frac{1+i^2+2i}{2}\right)^n = 1$

$\Rightarrow \left(\frac{1-1+2i}{2}\right)^n = 1$

$\Rightarrow i^n = 1$

The least the value of n is 4.

9. $(x+iy)^{\frac{1}{3}} = a + ib$

$\Rightarrow x + iy = (a+ib)^3$

$= a^3 + 3a^2ib + 3a.i^2b^2 + i^3b^3$

$= (a^3 - 3ab) + i(3a^2b - b^3)$.

Equaling the real and imaginary parts, we get

$x = a^3 - 3ab^2$

$y = 3a^2b - b^3$

$\therefore \frac{x}{a} = a^2 - 3b^2$ and $\frac{b}{b} = 3a^2 - b^2$

$\therefore \frac{x}{a} + \frac{y}{b} = (a^2 - 3b^2) + (3a^2 - b^2)$

$= 4a^2 - 4b^2$

$= 4(a^2 - b^2)$

10. Given that $\frac{a+ib}{c+id} = x + iy$ ... (1)

Taking the conjugate of both sides, we get

$\Rightarrow \overline{\left(\frac{a+ib}{c+id}\right)} = \overline{x+iy}$

$\Rightarrow \frac{\overline{a+ib}}{\overline{c+id}} = \overline{x+iy}$

$$\Rightarrow \frac{a-ib}{c-id} = x - iy \qquad .... (2)$$

Multiplying (1) and (2), we get

$$\frac{a+ib}{c+id} \text{ x } \frac{a-ib}{c-id} = (x+iy)(x-iy)$$

$$\Rightarrow \frac{a^2+b^2}{c^2+d^2} = x^2+y^2$$

11. Given equation is

$$9x^2 - 12x + 20 = 0$$

$$\Rightarrow (3x)^2 - 2.3x.2 + 4 + 24 = 0$$

$$\Rightarrow (3x-2)^2 = -24$$

$$\Rightarrow (3x-2)^2 = i^2.(2\sqrt{6})^2$$

$$\Rightarrow (3x-2)^2 - (2\sqrt{6}i)^2 = 0$$

$$\Rightarrow (3x - 2 + 2\sqrt{6}\ i)(3x - 2 - 2\sqrt{6}\ i = 0$$

$$\Rightarrow 3x = 2 - 2\sqrt{6}\,i,\ 2 + 2\sqrt{6}\,i$$

$$\Rightarrow x = \frac{2-2\sqrt{6}\,i}{3}, \frac{2+2\sqrt{6}\,i}{3}.$$

12. We know $1+\omega+\omega^2 = 0$

$\Rightarrow i+\omega = -\omega^2$ and $1+\omega^2 = -\omega$

L.H.S.

$$= (1-\omega+\omega^2)(1+\omega-\omega^2)$$

$$= (1+\omega^2-\omega)(1+\omega-\omega^2)$$

$$= (-\omega-\omega)(-\omega^2-\omega^2)$$

$$= -2\omega \text{ x } -2\omega^2 = 4\omega^3 = 4.$$

13. L.H.S.

$$= (1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)$$

$$= (1-\omega)(1-\omega^2)(1-\omega^3.\omega)(1-\omega^3.\omega^2)$$

$$= (1-\omega^2)(1-\omega^2)^2$$

$$= (1+\omega^2-2\omega)(1-\omega^4-2\omega^2) \quad (\because \omega^4 = \omega)$$

$$= -3\omega \text{ x } (-\omega^2-2\omega^2)$$

$$= -3\omega \text{ x } 3\omega^2$$

$$= 9^{\omega} = 9.$$

14. $(1+2-2\omega^2)^4(4+\omega+4\omega^2)^4$

$$= (-\omega^2-2\omega^2)[4(1+\omega^2)+\omega]^4$$

$$= -3\omega^2[-4\omega+\omega]^4$$

$$= -3\omega^2 \text{ x } -3\omega = 9\omega^3 = 9.$$

15. $\frac{a+ib}{a-ib} - \frac{a-ib}{a+ib}$

$$= \frac{(a+ib)^2-(a-ib)^2}{(a-ib)(a+ib)}$$

$$= \frac{2aib}{a^2+b^2} = 0 + i,\ \frac{2ab}{a^2+b^2}$$

16. Refer to text book.

17. $(1+a^2)(1+b^2)(1+c^2)$

$$= (1-i^2a^2)(1-i^2b^2)(1-i^2c^2)$$

$$= (1-ia)(1+ia)(1-ib)(1+ib)(1-ic)(1+ic)$$

$$= (1-ia)(1+ia)(1-ib)(1+ib)(1-ic)(1+ic)$$

$$= (1-ia)(1-ib)(1+ia)(1+ib)(1-ic)(1+ic)$$

$$= [1-i(a+b)+i^2ab][1+i(a+b)+i^2ab]$$
$$(1-ic)(1+ic)$$

$$= [1-ab-i(a+b)][1-ab+i(a+b)]$$
$$(1-ic)(1+ic)$$

$$= [1-ab-i(a+b)](1-ic)$$
$$[1-ab+i(a+b)](1+ic)$$

$$= [(1-ab-i(a+b)-ic+iabc+i^2c(a+b)]$$
$$\text{x}[1-ab+i(a+b)+ic-iabc+i^2c(a+b)]$$

$$= [(1-ab-bc-ac)+i(a+b+c-abc)$$
$$\text{x}[(1-ab-ac-bc)+i(a+b+c-abc)]$$

$$= (1-ab-bc-ac)^2 - i^2(a+b+c-abc)^2$$

$$= (1-ab-bc-ac)^2 + (a+b+c-abc)^2$$

18. $x^2-6x+25=0$

$$\Rightarrow x^2 - 2.x.3 + 9 + 16 = 0$$

$$\Rightarrow (x-3)^2 = -16 = (4i)^2$$

$$\Rightarrow x - 3 = \pm 4i$$

$$\Rightarrow x = 3 \pm 4i$$

19. $\dfrac{1}{(3+i)^2}+\dfrac{1}{(3-i)^2}=\dfrac{(3-i)^2+(3+i)^2}{(3+i)^2(3-i)^2}$

$=\dfrac{2(3^2+i^2)}{(9-i^2)^2}=\dfrac{2(9-1)}{10^2}$

$=\dfrac{2.8}{100}=\dfrac{4}{25}.$

20. Given that $x=1+i$

$\Rightarrow x-1=i$

$\Rightarrow (x-1)^2=i^2$

$\Rightarrow x^2-2x+1=-1$

$\Rightarrow x^2-2x+2=0$

$x^4-4x^3+7x^2-6x+3=$

$(x^4-2x^3+2x^2)+(-2x^3+4x^2-4x)+$

$(x^2-2x+2)+1$

$=x^2(x^2-2x+2)-2x(x^2-2x+2)$

$+(x^2-2x+2)+1$

$=x^2.0-2x.0+0+1=1$

21. Exercise to the students.

22. Exercise to the students.

23. Let P(n) be the statement "2^{3n}- 1 is a multiple of 7".

First we shall show that P(1) is true.

Taking n = 1, we get $2^{3.1}$-1 = 8-1 = 7 which is a multiple of 7.

So P(1) is true,

Let P (K) be true

$\Rightarrow 2^{3k}-1$ is a multiple of 7.

$\Rightarrow 2^{3k}-1=7m$ where $m\in Z$

We shall have to show that P(k+1) is true.

$\therefore 2^{3(k+1)}-1=2^{3k+3}-1=2^{3k}.\,2^3-1$

$=8.\,2^{3k}-1$

$=8(7m+1)-1$

$=8.7m+7$

$=7(8m+1)$ which is a multiple of 7.

So P(k+1) is true.

Here we see that

(i) P(1) is true

(ii) P(k) is true $\Rightarrow P(k+1)$ is true.

So according to the principle of induction, P(n) is true for all $n\in N$

$\Rightarrow 2^{3n}-1$ is a multiple of 7 for all $n\in N$.

24. $(1-i)^n\left(1-\dfrac{1}{i}\right)^n=\left[(1-i)\left(1-\dfrac{1}{i}\right)\right]^n$

$=1-i-\dfrac{1}{i}+1=2-\left(i+\dfrac{1}{i}\right)$

$=2-\dfrac{i^2+1}{i}=2-0=2$

25. The given equation is $\left|\dfrac{2z-i}{z+1}\right|=m$

$\Rightarrow \left|\dfrac{2\left(z-\dfrac{i}{2}\right)}{z+1}\right|=m$

$\Rightarrow 2\left|\dfrac{z-\dfrac{i}{2}}{z+1}\right|=m$

$\Rightarrow \left|\dfrac{z-\dfrac{i}{2}}{z+1}\right|=\dfrac{m}{2}$

This does not represent a circle when $\dfrac{m}{2}=1$ i.e. $m=2$.

26. The given inequation $3(x-1)\le 2(x-3)$

$\Rightarrow 3x-3\le 2x-6$

$\Rightarrow x\le -3$

The required solution set is $(\infty,\,-3)$.

27. The given in equation is

$x+\frac{x}{2}+\frac{x}{3}<11$

$\Rightarrow \frac{6x+3x+2x}{6}<11$

$\Rightarrow 11\,x<66$

The solution set is $(-\infty, 6)$.

28. The given inequation is $x+12<4x-2$

$\Rightarrow 4x-2>x+12$

$\Rightarrow 3x>14$

$\Rightarrow x>\frac{14}{3}$

The solution set is $\left(\frac{14}{3}, \infty\right)$.

29. Let x be the smallest of the two consecutive odd positive integrs. So other number is x+2.

Accoding to the question, x < 10.

$\therefore\ x+(x+2)>11$

$\Rightarrow 2\,x+2>11$

$\Rightarrow 2\,x>9$

$\Rightarrow x>\frac{9}{2}=4\frac{1}{2}$

Thus x can take the values 5, 7, 9.

Required possible pairs will be (5,7), (7,9).

30. Let the student set x marks in 3rd test.

According to the question, $\frac{70+75+x}{3}\geq 60$

$\Rightarrow 145+x\geq 180$

$\Rightarrow x\geq 35$

The student should set a minimum of 35 marks to set an average of at least 60 marks.

31. Given that the sets A and B have m and n elements respectively.

$\therefore\ |A|=m,\ |B|=n$

To form a function, we need to assocaite each element of A with exactly one element of B.

The first element in A can have images in B in n different ways. Following this, the 2nd element of A can have images in n different ways.

So first two elements of A can have images in B in n x n different ways.

Similarly the number of ways that m elements of A can have images in n x n x n x ... to m factors = n^m.

32. Given that P (n, 5)=20. P (n, 3)

$\Rightarrow n(n-1)(n-2)(n-3)(n-4)$

$= 20\ n(n-1)(n-2)$

$\Rightarrow (n-3)(n-4)=20$

$\Rightarrow n^2-7n+12=20$

$\Rightarrow n^2-7n-8=0$

$\Rightarrow n^2-8n+n-8=0$

$\Rightarrow n\,(n-8)+1\,(n-8)=0$

$\Rightarrow (n-8)(n+1)=0$

$\Rightarrow n=8,-1$

$\Rightarrow n=8$ (as n = -1 is rejected)

33. Give that $P(m+n, 2)=56$

$\Rightarrow (m+n)(m+n-1)=8.7$

$\Rightarrow m+n=8$ ... (1)

Again P (m - n, 2) = 12

$\Rightarrow (m-n)(m-n-1)=4 \text{ x } 3$

$\Rightarrow m-n=4$ ... (2)

Adding (1) and (2), we get

$2m=12 \Rightarrow m=6$

From (1), we get $6+n=8 \Rightarrow n=2$

34. L.H.S. = C (n, r) + C (n, r-1)

$=\frac{n!}{r!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)!}$

$=\frac{n!}{r\,(r-1)!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)(n-r)!}$

$=\frac{n!}{(r-1)!(n-r)!}\left\{\frac{1}{r}+\frac{1}{n-r+1}\right\}$

$$=\frac{n!}{(r-1)!(n-r)!}\ \frac{n-r+1+r}{r(n-r+1)}$$

$$=\frac{(n+1)n!}{r(r-1)!(n-r+1)(n-r)!}$$

$$=\frac{(n+1)!}{r!(n+1-r)!}=C(n+1,r).$$

35. Given that C(2n, r) = C(2n, r+2)

$\Rightarrow r+(r+2)=2n$

$\Rightarrow r+1=n$

$\Rightarrow r=n-1$

36. Among 15 acquintances of a gentlemen, 9 are relatives and 6 are non-relatives. He is to invite 8 quests so taht 6 of them are relatives and 2 are non-relatives.

6 guests are selected from 9 guests in C (9, 6) ways.

2 non relatives are selected from 6 non relatives in C(6, 2) ways.

Total number of selection = C (9, 6) x C(6, 2)

$$=\frac{9!}{6!\,3!}\ \text{x}\ \frac{6!}{2!\,4!}=1260$$

37. There are 8 gentlemen and 4 ladies. A committee of 5 is to be formed so as to include at least 1 lady.

The following types of selections are made.

1st type : 4 gents and 1 lady

2nd type : 3 gents and 2 ladies

3rd type : 2 gents and 3 ladies

4th type : 1 gentleman and 4 ladies.

1st type :

4 gents are selected from 8 gents in C (8, 4) ways.

1 lady is to be selected from 4 ladies in C(4,1) ways.

Total number of this type of committee

= C (8, 4) x C (4, 1)

$$=\frac{8!}{4!(8-4)!}\ \text{x}\ \frac{4!}{1!(4-1)!}$$

$$=\frac{8!}{4!\,4!}\ \text{x}\ \frac{4!}{1!\,3!}$$

$$=\frac{8.7.6.5}{4.3.2.1}\ \text{x}\ \frac{4}{1}=280$$

2nd type

Total number of 2nd type of committee

$= C(8,3)\ \text{x}\ C(4,2)$

$$=\frac{8!}{3!5!}\text{x}\frac{4!}{2!2!}$$

$$=\frac{8.7.6}{3.2.1}\text{x}\frac{4.3}{2!}$$

$= 56 \text{ x } 6 = 336$

3rd type

Total number of 3rd type of committee

= C(8, 2) x C(4, 3)

$$=\frac{8.7}{1.2}\text{x}\,4=112$$

4th type

Total number of 4th type of committee

= C (8, 1) x C(4, 4)= 8 x 1 = 8

Total number of committee

= 280 + 336 + 112 + 8 = 736.

38. The given expression is $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$

Let $(r+1)^{th}$ term be independent of x.

$\therefore\ (r+1)^{th}$ term

$$=C(9,r)\left(\frac{3}{2}x^2\right)^{9-r}.\left(-\frac{1}{3x}\right)^r$$

$$=(-1)^r\frac{3^{9-2r}}{2^{9-r}}C(9,3)\ x^{18-3r}.$$

Since this term is independent of x,

$18-3r=0$

$\Rightarrow 3r=18$

$\Rightarrow r=6$

So $(6+1)^{th}$ term or 7^{th} term is independent of x

$$7\text{th term} = (-1)^6.\frac{3^{9-12}}{2^{9-6}}C(9,3)x^0$$

$$=\frac{3^{-3}}{2^3}.\frac{9.8.7}{3.2.1}$$

$$=\frac{1}{8.27}.\frac{9.8.7}{3.2.1}=\frac{7}{18}.$$

39. The given expression is $\left(2x^2-\frac{1}{x}\right)^7$.

The number of term = 7 +1 = 8.

So there are two middle terms, 4th & 5th terms.

The 1st middle term = 4th term

$$=C(7,3).(2x^2)^{7-3}.\left(-\frac{1}{x}\right)^3$$

$$=(-1).C(7,3).(2x^2)^4.\ x^{-3}$$

$$=(-1).\frac{7.6.5}{3.2.1}.2^4.x^5$$

The 2nd middle term = 5^{th} term

$$=C(7,4)(2x^2)^{7-4}.\left(-\frac{1}{x}\right)^4$$

$$=\frac{7.6.5.4}{4.3.2.1}.2^3\ x^6.\frac{1}{x^4}$$

$$=280x^2$$

40. The given expression is $(1 + x)^{2n}$.

The number of terms = 2n+ 1

There is one middle term i.e. $(n+1)^{th}$ term.

The middle term = $(n+1)^{th}$ term.

$$=C(2n,n)x^n$$

$$=\frac{(2n)!}{n!n!}x^n$$

$$=\frac{1.2.3.4.5.6.\ ...\ (2n-1).2n}{n!n!}x^n$$

$$=\frac{1.3.5...\ (2n-1)}{n!}2^n.x^n$$

41. We know

$$C_0+C_1x+C_2x^2+C_3x^3+...+C_nx^n=(1+x)^n$$

Differentiating both sides, we get

$$\frac{d}{dx}(C_0+C_1x+C_2x^2+...+C_nx^n)=(1+x)^n$$

$$\Rightarrow C_1+2C_2x+3C_3x^2+...+nC_nx^{n-1}$$

$$=n(1+x)^{n-1}$$

Putting x = 1, we get

$$\Rightarrow C_1+2C_2+3C_3+...+nC_n$$

$$=n.(1+1)^{n-1}=n.2^{n-1}.$$

42. Let s be the 1st term and d be the common difference.

Given that

a = s + (p - 1)d

b = s + (q - 1)d

c = s + (r - 1)d

$\therefore (q-r)a+(r-p)b+(p-q)c$

= (q-r)[s + (p - 1)d] + (r - p)[s + (q - 1)d] +(p - q)]s + (r - 1)d]

= s(q - r) + s (r - p) + s (p - q) + (q - r)(p - 1)d + (r - p)(1 - r)d + (p - q)(r -1)d

= s. 0 + d . 0 = 0

43. Let a be 1^{st} term and d be the common difference.

Given that 10th term = 52

a + 9d = 52 ... (1)

Again 16^{th} term = 82

$\Rightarrow$ a + 15d = 82 ... (2)

Subtracting (1) from (2), we get

6d = 30

$\Rightarrow$ d = 5.

From (1), we get a + 9.5 = 52

$\Rightarrow$ a + 45 = 52

$\Rightarrow$ a = 52 - 45 = 7

30th term = 1 + 29d

= 7 + 29 x 5

= 7 + 145 = 152.

44. $$\sum_{n=1}^{\infty}\frac{n^2}{n!}=\sum_{n=1}^{\infty}\frac{n^2}{n(n-1)!}=\sum_{n=1}^{\infty}\frac{n}{(n-1)!}$$

$$=\sum_{n=1}^{\infty}\frac{(n-1)+1}{(n-1)!}$$

$$=\sum_{n=1}^{\infty}\frac{n-1}{(n-1)!}+\frac{1}{(n-1)!}$$

$$=\sum_{n=1}^{\infty}\frac{1}{(n-2)!}+\frac{1}{(n-1)!}$$

$$=\sum_{n=1}^{\infty}\frac{1}{(n-2)!}+\sum_{n=1}^{\infty}\frac{1}{(n-1)!}=e+e=2e$$

45. L.H.S. $\frac{1}{2}-\frac{1}{2.2^2}+\frac{1}{3.2^3}-\frac{1}{4.2^4}$

$$=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\ldots\text{where } x=\frac{1}{2}$$

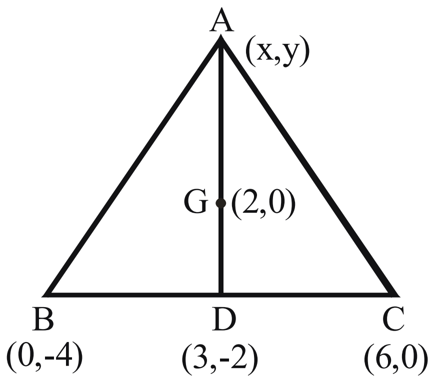
$$=\log_e(1+x)$$

$$=\log_e\left(1+\frac{1}{2}\right)=\log_3\left(\frac{3}{2}\right).$$

4. Straight lines, Conoc sections, Introduction to thre dimensional geometry.

1. Let ABC be a triangle. Let the coordinates of B and C be (0, -4) and (6, 0) respectively.

Let G be a point where three medians meet. Given that the coordinates of G are (2, 0).



Let AD be a median.

The coordinates of D are

$$\left(\frac{0+6}{2},\frac{-4+0}{2}\right)=(3,-2).$$

Let the coordinates of A be (x, y).

G divides AD in the ratio 2 :1.

$$\therefore 2=\frac{1.x+2.3}{3},\ 0=\frac{2(-2)+1.y}{3}$$

$$\Rightarrow 2=\frac{x+6}{3},\ 0=\frac{-4+y}{3}$$

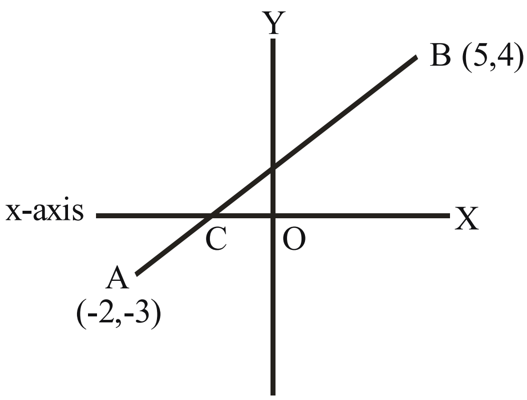
$$\Rightarrow x+6=6,\quad -4+y=0$$

$$\Rightarrow x=0,\quad y=4$$

The coordinates of A are (0, 4).

2. Let A and B be two points whose coordinates are (-2, -3) and (5, 4).

Let AB intersect at C. Let C divides AB in the ratio m : n.



The coordinates of C are $\left(\frac{5m-2n}{m+n}, \frac{4m-3n}{m+n}\right)$.

Since it is a point on x - axis, we have

$$\frac{4m-3n}{m+n}=0$$

$$\Rightarrow 4m-3n=0$$

$$\Rightarrow 4m=3n$$

$$\Rightarrow \frac{m}{n}=\frac{3}{4}$$

The line is divided by x-axis in the ratio 3:4.

3. Let A and B be two points whose coordiantes are (a+b, b-a) and (a-b, a+b) respectively. Let P be the point whose coordiantes are (x, y).

Given that PA = PB

$$\Rightarrow \sqrt{(a+b-x)^2+(b-a-y)^2}$$

$$=\sqrt{(a-b-x)^2+(a+b-b)^2}$$

$$\Rightarrow (a+b-x)^2+(b-a-y)^2$$

$$=(a-b-x)^2+(a+b-y)^2$$

$$\Rightarrow a^2+b^2+x^2+2ab-2ax-2bx+$$

$$b^2+a^2+y^2-2ab-2by+2ay$$

$$=a^2+b^2+x^2-2ab-2ax+2bx+$$

$$a^2+b^2+y^2+2ab-2ab-2by$$

$$\Rightarrow -4bx=-4ay$$

$$\Rightarrow bx-ay=0$$

4. Let A and B be two points whose coordinates are (3, -4) and (1,2) respectively. Let C be the middle point of AB.

The coordinates of C are

$$\left(\frac{3+1}{2}, \frac{-4+2}{2}\right)=(2,-1).$$

Slope of the line $AB=\frac{2-(-4)}{1-3}=\frac{6}{-2}=-3$

Let the slope of the line passing through C and perpendicular to AB be m.

$$\therefore\ mx-3=-1 \quad \Rightarrow m=\frac{1}{3}.$$

The equation the line passing through C and perpendicular to AB is

$$y-(-1)=\frac{1}{3}(x-2)$$

$$\Rightarrow y+1=\frac{1}{3}(x-2)$$

$$\Rightarrow 3y+3=x-2$$

$$\Rightarrow x-y=5$$

5. Let P be the given point (-4, 2).

The given line is 4x - 3y = 10 ... (1)

Slope of the given line $=-\frac{4}{(-3)}=\frac{4}{3}$.

Since the required line is parallel to the given line, its slope $=\frac{4}{3}$.

The equation of the line is

$$y-2=\frac{4}{3}(x+4)$$

$$\Rightarrow 3y-6=4x+16$$

$$\Rightarrow 4x-3y+22=0$$

6. Three given line are

A(h, o), B(a, b) and C (o, k).

$$m_{AB}=\frac{b-0}{a-h}=\frac{b}{a-h}$$

$$m_{AC}=\frac{k-0}{a-h}=-\frac{k}{h}$$

Since the three points lies on a line, we have

$$m_{AB}=m_{AC}$$

$$\Rightarrow \frac{b}{a-h}=-\frac{k}{h}$$

$$\Rightarrow bh=-k(a-h)$$

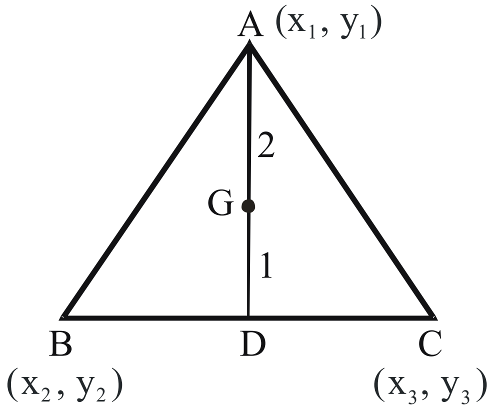
$$\Rightarrow bh=-ak+hk$$

$$\Rightarrow ak+bh=hk$$

$$\Rightarrow \frac{a}{h}+\frac{b}{k}=1$$

7. Let ABC be a triangle. Let the coordinates of A, B and C be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively.

Let D be the middle point of BC.



The coordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$.

Let G be the centroid of the triangle ABC.

G divides AD in the ratio 2 : 1.

The coordinates of G are

$$\left[\frac{1.x_1+2\left(\frac{x_2+x_3}{2}\right)}{1+2}, \frac{1.y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2}\right]$$

$$=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

8. Let A, B, C and D be the points whose coordinates are (1, 1), (-2, 4), (-1, 2) and (2, -1) respectively.

We shall show that ABCD is a parallelogram.

$$m_{AB} = \text{Slope of AB} = \frac{4-1}{-2-1} = \frac{3}{-3} = -1$$

$$m_{DC} = \text{Slope of DC} = \frac{-1-2}{2-(-1)} = \frac{-3}{3} = -1$$

$m_{AB} = m_{DC} \Rightarrow$ AB is parallel to DC.

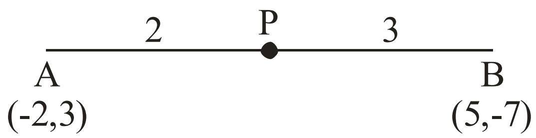
Similarly $m_{BC} = m_{AD} \Rightarrow$ BC is parallel to AD.

So ABCD is a parallelogram.

9. An exercise to the students.

10. Let A and B be two points whose coordinates are (-2, 3) and (5, -7) respectively.

Let P be a point on AB which divides AB in the ratio 2:3 internally.



The coordinates of P are

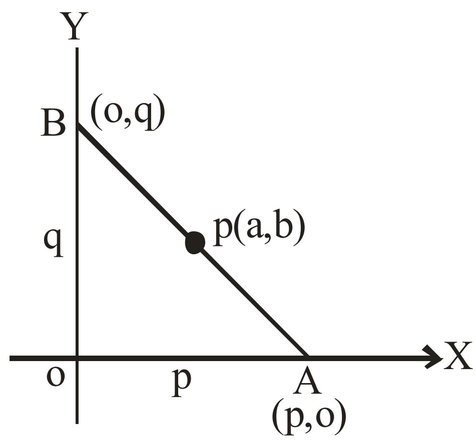
$$\left(\frac{2.5+3(-2)}{2+3}, 2\frac{(-7)+3.3}{2+3}\right)$$

$$=\left(\frac{10-6}{5}, \frac{-14+9}{5}\right)$$

$$=\left(\frac{4}{5}, -1\right).$$

11. Let P(a,b) be the middle point of the line segment AB.

Let OA = p, OB = q.



The coordinates of A are (p, o) and that of B are (o, q).

$$\therefore a = \frac{p+o}{2} = \frac{p}{2}$$

$$b = \frac{q+o}{2} = \frac{q}{2}$$

$$\Rightarrow p = 2a, q = 2b.$$

The equation of the line is

$$\frac{x}{p}+\frac{y}{q}=1$$

$$\Rightarrow \frac{x}{2a}+\frac{y}{2b}=1$$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}=2.$$

12. Three given lines are

$2x - 3y + 4 = 0$ ... (1)

$2x + y - 3 = 0$ ... (2)

$x + 2y - 3 = 0$ ... (3)

The equation of the line passing through (2) and (3) is

$2x + y - 3 + k (x + 2y - 3) = 0$

$\Rightarrow 2x + y - 3 + kx + 2ky - 3k = 0$

$\Rightarrow (2+k) x + (1+2k)y-(3+3k)=0$ ... (4)

Slope of this line $= -\left(\frac{2+k}{1+2k}\right)$

Slope of the line (1) $= -\frac{2}{(-3)} = \frac{2}{3}$

Since the line (4) is perpendicular to the line (1), we have

$$\frac{2}{x}x - \frac{(2+k\_}{1+2k} = -1$$

$$\Rightarrow 2(2+k) = 2(1-2k)$$

$$\Rightarrow 4+2k = 3(1+2k)$$

$$\Rightarrow 4+2k = 3+6k$$

$$\Rightarrow 4k = 1$$

$$\Rightarrow k = \frac{1}{4}.$$

The required line is

$$2x+y-3+\frac{1}{4}(x+2y-3)=0$$

$$\Rightarrow 8x+4y-12+x+2y-3=0$$

$$\Rightarrow 9x+6y-15=0$$

$$\Rightarrow 3x+2y-5=0$$

13. The given lines are

$x + y + 1 = 0$ ... (1)

$2x - y - 3 = 0$ ... (2)

Slope of the line (1) $= -\frac{1}{1} = -1$

Since the required line is parallel to the line (1), slope of the required line = -1.

y - intercept of the line (2) $= -\frac{(-3)}{-1} = -3$.

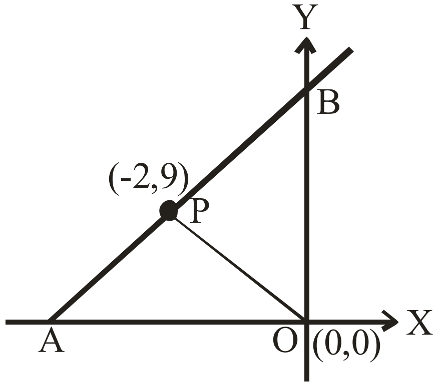
Thus the slope of the required line is -1 and its y-intercept is -3.

The required equation is $y = mx + c$

$$\Rightarrow y = (-1)x + (-3)$$

$$\Rightarrow x + y + 3 = 0$$

14. Let AB be the given line. Let OP be the perpendicular to the line AB. The coordinates of P as (-2, 9).



Slope of the line

$$OP = \frac{9-0}{-2-0} = -\frac{9}{2}$$

$$\Rightarrow m_{OP} = -\frac{9}{2}$$

Since AB is perpendicular to OP, we have

$$m_{AB} \text{ x } m_{OP} = -1$$

$$\Rightarrow m_{AB} x - \frac{9}{2} = -1$$

$$\Rightarrow m_{AB} = \frac{2}{9}$$

AB is a line passing through (-2, 9) and whose slope is $\frac{2}{9}$.

Equation of the line AB is

$$y-9=\frac{2}{9}(x+2)$$

$$\Rightarrow 9y-81=2x+4$$

$$\Rightarrow 2x-9y+85=0.$$

15. Two given lines are

x - 2y - 4 = 0 ... (1)

y - 2x - 3 = 0 ... (2)

$$\text{x intercept of (1)} = \frac{-(-4)}{1}=-4$$

$$\text{y intercept of the line (2)} = \frac{-(-3)}{1}=-4$$

x - intercept of the required line = -4 and its y-intercept - 3.

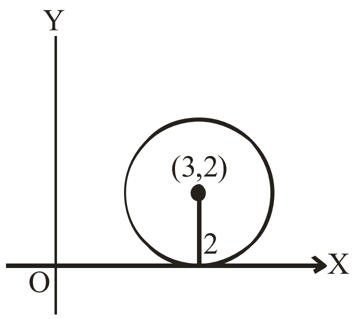
The equation of the line is $\frac{x}{-4}+\frac{y}{3}=0$

$$\Rightarrow \frac{-3x+4y}{12}=0$$

$$\Rightarrow -3x+4y=12$$

$$\Rightarrow -3x-4y+12=0$$

16. Centre of the cirlce is (3, 2). Since the circle is tangent to x-axis, its radius is 2.



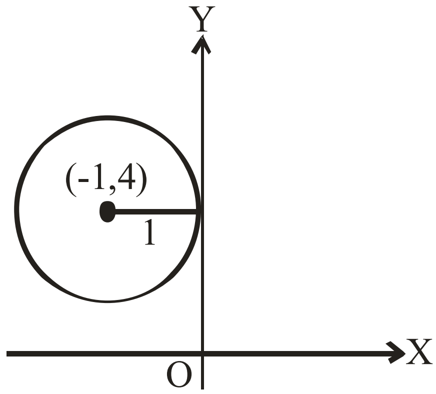
The equation of the circle is

$$(x-3)^2+(y-2)^2=2^2$$

$$\Rightarrow x^2-6x+9+y^2-4y+4=4$$

$$\Rightarrow x^2+y^2-6x-4y+9=0.$$

17. The centre of the cirlce is (-1, 4). Since the circle is tangent to y-axis, its radius is 1.



The equation of the cirlce is

$$(x+1)^2+(y-4)^2=1^2$$

$$\Rightarrow x^2+2x+1+y^2-8y+16=1$$

$$\Rightarrow x^2+y^2+2x-8y+16=0$$

18. The given line is

2x + y - 3 = 0 ... (1)

Let A and B be two points on the circle whose coordinates are (5, 1) and (2, -3). Let C be the centre whose coordinates are (h, k).

Since the centre is on the line (1), we have

2h + k - 3 = 0 ... (2)

Also CB = CA

$$\Rightarrow \sqrt{(h-2)^2+(k+3)^2}=\sqrt{(h-5)^2+(k-1)^2}$$

$$\Rightarrow (h-2)^2+(k+3)^2=(h-5)^2+(k-1)^2$$

$$\Rightarrow h^2+4h+4+k^2+6k+9$$

$$=h^2-10h+25+k^2-2k+1$$

$$\Rightarrow -4h+6k+13=-10h-2k+26$$

$$\Rightarrow 6h+8k-13=0 \quad ...(3)$$

Solving (2) & (3), by cross multiplication, we get

$$h=\frac{11}{10},\ k=\frac{8}{10}.$$

Centre is $\left(\frac{11}{10}, \frac{8}{10}\right)$

Radius $CA = \sqrt{\left(5-\frac{11}{10}\right)^2 + \left(1-\frac{8}{10}\right)^2}$

$= \sqrt{\left(\frac{39}{10}\right)^2 + \left(\frac{2}{10}\right)^2}$

$= \sqrt{\frac{1521+4}{100}} = \sqrt{\frac{1525}{100}}$.

Equation of the circle is

$$\left(x-\frac{11}{10}\right)^2 + \left(y-\frac{8}{10}\right)^2 = \frac{1525}{100}.$$

19. The given equation is

$4x^2 + 4y^2 - 4x + 12y - 15 = 0$

$\Rightarrow x^2 + y^2 - x + 3y - \frac{15}{4} = 0$

$\Rightarrow x^2 - 2x.\frac{1}{2} + \frac{1}{4} + y^2 + 2.y.\frac{3}{2} + \frac{9}{4} - \frac{1}{4} - \frac{9}{4} - \frac{15}{4} = 0$

$\Rightarrow \left(x-\frac{1}{2}\right)^2 + \left(y+\frac{3}{2}\right)^2 = \frac{25}{4}$

$\Rightarrow \left(x-\frac{1}{2}\right)^2 + \left[y-\left(-\frac{3}{2}\right)\right]^2 = \left(\frac{5}{2}\right)^2.$

The centre of the circle is $\left(\frac{1}{2}, -\frac{3}{2}\right)$ and radius $= \frac{5}{2}$.

20. Three given points are (0, 0), (0, 2) and (-1, 0).
Let the equation of the cirlce be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (1)

Since it is passing through (0, 0), we have

$0^2 + 0^2 + 2g.0 + 2f.0 + c = 0$

$\Rightarrow c = 0$

Since it is passing through (0, 2), we have

$0^2 + 4 + 2g.0 + 2t.\,2 + c = 0$

$\Rightarrow 4f = -4 \Rightarrow f = -1$.

Since the circe is passing through (-1, 0), we have

$(-1)^2 + 0^2 + 2g(-1) + 2f.0 + 0 = 0$

$\Rightarrow 1 - 2g = 0$

$\Rightarrow 2g = 1 \Rightarrow g = \frac{1}{2}$.

Equation of the circle is

$x^2 + y^2 + 2.\frac{1}{2}.x + 2(-1).y + 0 = 0$

$\Rightarrow x^2 + y^2 + x - 2y = 0$

21. Vertex is (2, 3).
Focus is (-2, 3)
a = - 2 - 2 = - 4.
Equation of the parabola is

$(y-3)^2 = 4.(-4)(x-2)$

$\Rightarrow (y-3)^2 = -16(x-2)$

22. Three points on the parabola are (1, 2), (-2, 3) and (2, -1).
Axis of the parabola is parallel to x-axis.
Let the equation of the parabola be

$y^2 + ax + by + c = 0$ ... (1)

Since it is passing through (1, 2), we have

$2^2 + a.1 + b.2 + c = 0$

$\Rightarrow a + 2b + c + 4 = 0$ ... (2)

Similarly $3^2 + a(-2) + b.3 + c = 0$

$\Rightarrow -2a + 3b + c + 9 = 0$ ... (3)

and $(-1)^2 + a.2 + b(-1) + c = 0$

$\Rightarrow 2a - b + c + 1 = 0$ ... (4)

Subtracting (2) from (3), we get

$-2a - a + 3b - 2b + 9 - 4 = 0$

$\Rightarrow -3a + b + 5 = 0$ ... (5)

Subtracting (4) from (2), we get

$a - 2a + 2b + b + 3 = 0$

$\Rightarrow -a + 3b + 3 = 0$ ... (6)

From (5) and (6) by cross multiplication,

$$\frac{a}{3-15} = \frac{b}{-5+9} = \frac{1}{-9+1}$$

$$\Rightarrow \frac{a}{-12} = \frac{b}{4} = \frac{1}{-8}$$

$$\Rightarrow a = -\frac{12}{-8} = \frac{3}{2},\ b = -\frac{4}{8} = -\frac{1}{2}.$$

From (2), we get

$$\frac{3}{2} + 2\left(-\frac{1}{2}\right) + c + 4 = 0$$

$$\Rightarrow \frac{3}{2} - 1 + c + 4 = 0$$

$$\Rightarrow \frac{1}{2} + c + 4 = 0$$

$$\Rightarrow c + \frac{9}{2} = 0$$

$$\Rightarrow c = -\frac{9}{2}$$

The required equation of the parabola is

$$y^2 + \frac{3}{2}x - \frac{1}{2}y - \frac{9}{2} = 0$$

$$\Rightarrow 2y^2 + 3x - y - 9 = 0$$

23. The focus of the parabola is F(1, 2). Equation of the directrix is x + y - 2 = 0 ... (1)

Let P be a point on the parabola whose coordinates are (x, y).

From P, let us draw a perpendicular PM to the directrix.

PF = PM

$$\Rightarrow \sqrt{(x-1)^2 + (y-2)^2} = \frac{x+y-2}{\sqrt{1^2+1^2}}$$

$$\Rightarrow (x-1)^2 + (y-2)^2 = \frac{(x+y-2)^2}{2}$$

$$\Rightarrow 2[(x-1)^2 + (y-2)^2] = (x+y-2)^2$$

$$\Rightarrow x^2 + y^2 - 2xy - 4y + 6 = 0$$

24. Vertex of the parabola is (h, k) = (6, -2).

Focus is (a + h, k) = (-3, 2).

Axis is parallel to x - axis.

h = 6, k = -2

a + h = -3 $\Rightarrow$ a + 6 = -3 $\Rightarrow$ a = - 9.

Parabola opens to the left.

Equation of the parabola is

$(y - k)^2 = 4a\,(x - h)$

$\Rightarrow (y + 2)^2 = 4\,(-9)\,(x - 6)$

$\Rightarrow (y + 2)^2 = -36\,(x - 6).$

25. Axis of the parabola is vertical i.e. the axis is parallel to y-axis.

Let the equation of the parabola be

$x^2 + ax + by + c = 0$ ... (1)

The parabola is passing through (0, 2), (-1, 1) and (2, 10).

Since (0, 2) is a point on the parabola (1), we get

$2^2 + a\,.\,0 + b\,.\,2 + c = 0$

$\Rightarrow 2b + c + 4 = 0$ ... (2)

Similarly

$(-1)^2 + a.\ 2 + b\ .\ 10 + c = 0$

$\Rightarrow\ - a + b + c + 1 = 0$ ... (3)

$2^2 + a\ .\ 2 + b.\ 10 + c = 0$

$\Rightarrow\ 2a + 10\ b + c + 4 = 0$ ... (4)

Solving (2), (3) & (4), we get

$a = -4,\ b = 1,\ c = -6$

Equation of the parabola is

$x^2 - 4x + y - 6 = 0.$

26. Centre of the ellipse is (0, 0).

One vertex is (0, -5).

So other vertex is (0, 5).

Major axis is along y-axis.

$\therefore\ a = 5.$

$\therefore\ b = 3$

Equation of the ellipse is

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

$$\Rightarrow \frac{y^2}{5^2} + \frac{x^2}{3^2} = 1$$

$$\Rightarrow \frac{y^2}{25} + \frac{x^2}{9} = 1.$$

27. The foci of the ellipse is $(\pm 5, 0)$

i.e. $(\pm c, 0) = (\pm 5, 0)$

$\therefore c = 5$

Length of the major axis = 12

$\Rightarrow 2a = 12$

$\Rightarrow a = 6\cdot$

$\therefore b^2 = a^2 - c^2 = 36 - 25 = 11\cdot$

Major axis is on x - axis.

Equation of the ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$$\Rightarrow \frac{x^2}{36} + \frac{y^2}{11} = 1.$$

28. The vertices of the ellipse are

$(\pm a, 0) = (\pm 5, 0)$

$\therefore\ a = 5.$

Major axis is one x - axis.

Length of latus rectum $= \dfrac{8}{5}$.

i.e. $2\dfrac{b^2}{a} = \dfrac{8}{5}$

$$\Rightarrow 2\frac{b^2}{5} = \frac{8}{5} \Rightarrow b^2 = 4$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{25} + \frac{y^2}{4} = 1.$$

29. Centre of the ellipse is (5, 4)

i.e. (h, k) = (5, 4).

Length of the major axis = 16

$\Rightarrow 2a = 16 \Rightarrow a = 8$

Length of the minor axis = 10

$\Rightarrow 2b = 10 \Rightarrow b = 5$

If the major axis is parallel to x-axis, then the equation of the ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x-5)^2}{64} + \frac{(y-4)^2}{25} = 1$$

30. Let the equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{... (1)}$$

Since it is passing through (-3, 2), we have

$$\frac{(-3)^2}{a^2} + \frac{2^2}{b^2} = 1$$

$$\Rightarrow \frac{9}{a^2} + \frac{4}{b^2} = 1 \quad \text{... (2)}$$

Since it is passing through (-5, 1), we have

$$\Rightarrow \frac{(-5)^2}{a^2}+\frac{1^2}{b^2}=1$$

$$\Rightarrow \frac{25}{a^2}+\frac{1}{b^2}=1 \qquad \text{... (3)}$$

Solving (2) and (3), we get

$$a^2=\frac{91}{3},\ b^2=\frac{91}{16}.$$

Equation of the ellipse is

$$\frac{x^2}{\frac{91}{3}}+\frac{y^2}{\frac{91}{16}}=1$$

$$\Rightarrow 3x^2+16y^2=91.$$

31. The distance between two foci is 16

$\Rightarrow$ 2 c = 16

$\Rightarrow$ c = 8

Eccentricity $=\sqrt{2}$

$$\Rightarrow \frac{c}{a}=\sqrt{2}$$

$$\Rightarrow \frac{8}{a}=\sqrt{2}$$

$$\Rightarrow a=\frac{8}{\sqrt{2}}=4\sqrt{2}.$$

Again $c^2 = a^2 + b^2$

$$\Rightarrow 8^2=(4\sqrt{2})^2+b^2$$

$$\Rightarrow 64=32+b^2$$

$$\Rightarrow b^2=64-32=32$$

$$\Rightarrow b=4\sqrt{2}.$$

Equation of the hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\Rightarrow \frac{x^2}{(4\sqrt{2})^2}-\frac{y^2}{4\sqrt{2})^2}=1$$

$$\Rightarrow x^2-y^2=32.$$

32. The foci of the hyperbola are $(\pm 4, 0)$

i.e. $(\pm c, 0)=\pm 4$

$\Rightarrow c=4.$

The transverse axis is along x-axis vertices are at $(\pm 2, 0)$

$$\Rightarrow (\pm a, 0)=(\pm 2, 0)$$

$$\Rightarrow a=2$$

$$\therefore b^2=c^2-a^2=4^2-2^2=16-4=12$$

Equation of the hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\Rightarrow \frac{x^2}{4}-\frac{y^2}{12}=1.$$

33. Centre of the hyperbola is at (0, 0).

Length of the transverse axis = 4

$\Rightarrow$ 2a = 4 $\Rightarrow$ a = 2

Fouces is at $(2\sqrt{5}, 0)$

$$\Rightarrow c=2\sqrt{5}.$$

$$b^2=c^2-a^2=(2\sqrt{5})^2-2^2$$

$$=20-4=16$$

Since the transverse axis is along x-axis, the equation of the hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\Rightarrow \frac{x^2}{2^2}-\frac{y^2}{16}=1$$

$$\Rightarrow \frac{x^2}{4}-\frac{y^2}{16}=1$$

34. The equation of the hyperbola is

$9x^2 - 16y^2 = 144$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$

$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

$\therefore a = 4,\ b = 3$

$\therefore c^2 = a^2 + b^2 = 4^2 + 3^2 = 16 + 9 = 25$

$\Rightarrow c = 5$.

Centre is (0, 0)

Foci are at $(\pm c, 0) = (\pm 5, 0)$

Vertices are at $(\pm 9, 0) = (\pm 4, 0)$

eccentricity $= \frac{c}{a} = \frac{5}{4}$.

Length of the latus recturm $= 2\frac{b^2}{a}$

$= 2.\frac{3^2}{4} = \frac{18}{4} = \frac{9}{2}$.

35. The foci of the hyperbola are at $(\pm 2\sqrt{3},\ 0)$,

$\therefore c = 2\sqrt{3}$

The transverse axis is along x-axis,

eccentricity $e = \frac{c}{a}$

$\Rightarrow \sqrt{3} = \frac{2\sqrt{3}}{a} \quad \Rightarrow a = 2$

$b^2 = c^2 - a^2 = (2\sqrt{3}) - 2^2$

$= 12 - 4 = 8$

$\Rightarrow b = 2\sqrt{2}$

The equation of the hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{2^2} - \frac{y^2}{(2\sqrt{2})^2} = 1$

$\Rightarrow \frac{x^2}{4} - \frac{y^2}{8} = 1$

36. Let ABC be a triangle whose vertices are (0, 1, 2), (2, 0, 4) and (-4, -2, 7) respectively

$AB = \sqrt{(2-0)^2 + (0-1)^2 + (4-2)^2}$

$= \sqrt{4+1+4} = \sqrt{9} = 3$

$BC = \sqrt{(-4-2)^2 + (-2-0)^2 + (7-4)^2}$

$= \sqrt{36+4+9} = \sqrt{49} = 7$

$AC = \sqrt{(-4-0)^2 + (-2-1)^2 + (7-2)^2}$

$= \sqrt{16+9+25} = \sqrt{50} = 5\sqrt{2}$.

Perimetr of the trianlge i.e. -

$3 + 7 + 5\sqrt{2} = 10 + 5\sqrt{2}$ unit.

37. Let A, B, C be three points whose coordinates are (a, b, c), (b, c, a) and (c, a, b) respectively

$AB = \sqrt{(b-a)^2 + (c-b)^2 + (a-c)^2}$

$= \sqrt{b^2 + a^2 - 2ab + b^2 + c^2 - 2ab + a^2 + c^2 - 2ac}$

$= \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac}$

$BC = \sqrt{(c-b)^2 + (a-c)^2 + (b-a)^2}$

$= \sqrt{c^2 + b^2 - 2bc + a^2 + c^2 - 2ac + b^2 + a^2 - 2ab}$

$= \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac}$

$BC = \sqrt{(a-c)^2 + (b-a)^2 + (c-b)^2}$

$= \sqrt{a^2 + c^2 - 2ac + b^2 + a^2 - 2ab + c^2 + b^2 - 2bc}$

$= \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac}$

$\therefore AB = BC = CA$

$\Rightarrow ABC$ in an equilateral triangle.

38. Let A, B, C be three points whose coordinaes are (2, 3, 2), (5, 5, 6) and (-4, -1, -6) respectively.

$$AB=\sqrt{(5-2)^2+(5-3)^2+(6-2)^2}$$

$$=\sqrt{9+4+16}=\sqrt{29}$$

$$BC=\sqrt{(-4-5)^2+(-1-5)^2+(-6-6)^2}$$

$$=\sqrt{81+36+144}=\sqrt{261}=3\sqrt{29}$$

$$AC=\sqrt{(-4-2)^2+(-1-3)^2+(-6-2)^2}$$

$$=\sqrt{36+16+64}$$

$$=\sqrt{116}=2\sqrt{29}\cdot$$

Here we see that AB + AC = BC

So the three points are collinear.

39. Let A and B be two points whose coordinates are (1, 2, 3) and (3, 2, -1) respectively.

Let P be a point equidistant from A & B

$$\therefore PA = PB$$

$$\Rightarrow\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$$

$$=\sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$$

$$\Rightarrow x^2-2x+1+y^2-4y+4+z^2-6z+9$$

$$=x^2-6x+9+y^2-4y+4+z^2+2z+1$$

$$\Rightarrow -2x-4y-6z+14=6x-4y-2z+14$$

$$\Rightarrow 4x-8z=0$$

$$\Rightarrow x-2z=0\cdot$$

40. Let A and B be two points whose coordinates are (1, 3, -1) and (2, 6, -2) respectively.

Let zx - plane divides AB at C in the ratio k :1.

The coordinates of C are

$$\left(\frac{2k+1}{k+1},\frac{6k+3}{k+1},\frac{-2k-1}{k+1}\right)$$

Since it is a point on zx - plane,

we have $\frac{6k+3}{k+1}=0$

$$\Rightarrow 6k+3=0$$

$$\Rightarrow 6k=-3$$

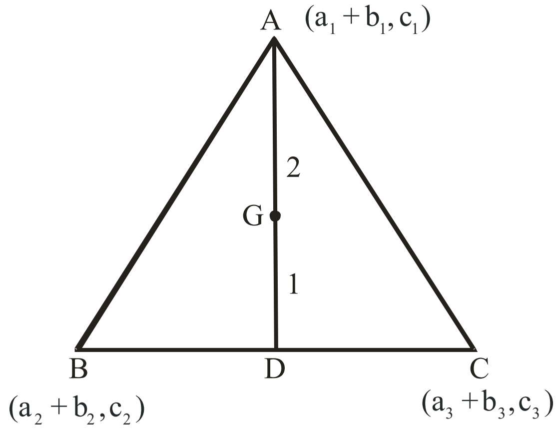
$$\Rightarrow k=-\frac{3}{6}=-\frac{1}{2}.$$

zx - plane divides AB in the ratio - 1 : 2.

41. Let ABC be a triangle. Let the B(a_2, b_2, c_2) and c(a_3, b_3, c_3) be the vertices.

Let D be the middle point of BC.

The coordinates of D are



$$\left(\frac{a_2+a_3}{2},\frac{b_2+b_3}{2},\frac{c_2+c_3}{2}\right)$$

Let G be the centroid which divides AP in the ratio 2 : 1.

The coordinates of G are

$$\left(\frac{1.a_1+2(\frac{a_2+a_3}{2})}{1+2},\ \frac{1.b_1+2.(\frac{b_2+b_3}{2})}{1+2},\right.$$

$$\left.\frac{1.c_1+2.(\frac{c_2+c_3}{2})}{1+2}\right)$$

$$=\left(\frac{a_1+a_2+a_3}{3},\frac{b_1+b_2+b_3}{3},\frac{c_1+c_2+c_3}{3}\right)$$

42. Let A, B, C be three points whose coordinates are (0, 7, -10), (1, 6, -6) and (4, 9, -6) respectively.

$$AB = \sqrt{(0-1)^2 + (7-6)^2 + (-10+6)^2}$$

$$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2}$$

$$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(4-0)^2 + (9-7)^2 + (-6+10)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36} = 6$$

Here $AB = BC = 3\sqrt{2}$

So the Δ ABC is an isosceles triangle.

43. Let A and B be two points whose coordiantes are (-2, 3, 5) and (1, -4, 6).

Let P be a point on AB which divides it in the ratio 2 : 3 externally.

The coordinates of P are

$$= \left(\frac{2.1-3(-2)}{2-3}, \frac{2(-4)-3.3}{2-3}, \frac{2.6-3.5}{2-3}\right)$$

$$= (-8, 17, 3)$$

44. Let P and Q be two points (-2, 4, 7) and (3, -5, 8) respectively. Let PQ intersect yz-plane at R in the ratio k : 1.

The coordinates of R are

$$\left(\frac{k.3-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k.8+7}{k+1}\right)$$

$$= \left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1}\right)$$

Since this point is on yz - plane,

We have $\dfrac{3k-2}{k+1} = 0$

$$\Rightarrow 3k-2=0 \qquad \Rightarrow 3k = 2 \qquad \Rightarrow k = \frac{2}{3}$$

Thus yz-plane divides PQ in the ratio 2 :3.

45. Let A, B & C be three points whose coordinates are (0, 7, 10), (-1, 6, 6) and (-4, 9, 6).

$$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$$

$$= \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(-1+4)^2 + (9-6)^2 + (6-6)^2}$$

$$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36} = 6$$

Here we see that $AB^2 + BC^2 = AC^2$

$\Rightarrow$ ABC is a right angle triangle.

5. Limits and Derivatives

1.(i) $\lim_{x\to 2^-}[x]=1$

$\lim_{x\to 2^+}[x]=2$

$\therefore \lim_{x\to 2^-}[x]\neq \lim_{x\to 2^+}[x]$

So $\lim_{x\to 2}[x]$ does not exist.

(ii) Given limit is $\lim_{x\to 4} x-[x]$.

L.H.L. $=\lim_{x\to 4^-} x-[x]$

$=\lim_{x\to 4^-} x-3=4-3=1$

R.H.L. $=\lim_{x\to 4^+} x-[x]$

$=\lim_{x\to 4^+} x-4=4-4=0$

$\therefore \lim_{x\to 4^-} x-[x]\neq \lim_{x\to 4^+} x-[x]$

So $\lim_{x\to 4} x-[x]$ does not exist.

(iii) $\lim_{x\to 3}\frac{|x-3|}{x-3}$

We know $|x-3|=\begin{cases} x-3 & \text{when } x\geq 3 \\ -(x-3) & \text{when } n<3\end{cases}$

L.H.L. $=\lim_{x\to 3^-}\frac{|x-3|}{x-3}=\lim_{x\to 3^-}-\frac{(x-3)}{x-3}=-1$.

R.H.L. $=\lim_{x\to 3^+}\frac{|x-3|}{x-3}$

$=\lim_{x\to 3^+}\frac{x-3}{x-3}=1$

$\therefore \lim_{x\to 3^-}\frac{|x-3|}{x-3}\neq \lim_{x\to 3^+}\frac{|x-3|}{x-3}$

$\Rightarrow \lim_{x\to 3}\frac{|x-3|}{x-3}$ does not exit.

(iv) The given limit is $\lim_{x\to\sqrt{2}}[x]+3$.

L.H.L. $\lim_{x\to\sqrt{2}}[x]+3$ =1 + 3 = 4

R.H.L. $\lim_{x\to\sqrt{2}}[x]+3=1+3=4$

$\therefore \lim_{x\to\sqrt{2}^-}[x]+3=\lim_{x\to\sqrt{2}^+}[x]+3=4$

So $\lim_{x\to\sqrt{2}}[x]+3$ exists and equal to 4.

2. $\lim_{x\to\sqrt{2}}\frac{x^3-8}{x^5-32}$

$=\lim_{x\to 2}\frac{x^3-2^3}{x^5-2^5}$

$=\lim_{x\to 2}\frac{\frac{x^3-2^3}{x-2}}{\frac{x^5-2^5}{x-2}}$

(Deividing x-2 both in the numerator and denominator).

$=\frac{\lim_{x\to 2}\left(\frac{x^3-2^3}{x-2}\right)}{\lim_{x\to 2}\left(\frac{x^5-2^5}{x-2}\right)}$

$=\frac{3.2^2}{5.2^4}=\frac{3.4}{5.16}=\frac{3}{20}$.

3. $\lim_{x\to 0}\frac{(3+x)^3-27}{x}$

$=\lim_{x\to 0}\frac{(3+x)^3-3^3}{x}$

$=\lim_{x\to 0}\frac{(3+x-3)(3+x+3)}{x}$

$=\lim_{x\to 0}\frac{x(6+x)}{x}$

$=\lim_{x\to 0}6+x=6$

4. $\lim_{x\to 1}\frac{x^m-1}{x^n-1}$

$=\lim_{x\to 1}\frac{x^m-1^m}{x^n-1^n}$

$=\lim_{x\to 1}\frac{\left(\frac{x^m-1^m}{x-1}\right)}{\left(\frac{x^n-1^n}{x-1}\right)}$

(Dividing x-1 both in the numerator and denomination.)

$=\frac{\lim_{x\to 1}\left(\frac{x^m-1^m}{x-1}\right)}{\lim_{x\to 1}\left(\frac{x^n-1^n}{x-1}\right)}=\frac{m.1^{m-1}}{n.1^{n-1}}=\frac{m}{n}$.

5. $\lim_{x\to\infty}\frac{n^2+n+1}{5n^2+2n+1}$

$=\lim_{x\to\infty}\frac{\frac{n^2+n+1}{n^2}}{\frac{5n^2+2n+1}{n^2}}$

$=\frac{\lim_{x\to\infty}\left(1+\frac{1}{n}+\frac{1}{n^2}\right)}{\lim_{x\to\infty}\left(5+\frac{2}{n}+\frac{1}{n^2}\right)}=\frac{1}{5}$.

6. $\lim_{x\to\infty}\frac{1+2+3+...+n}{n^2}$

$=\lim_{x\to\infty}\frac{\frac{n(n+1)}{2}}{n^2}$

$=\lim_{x\to\infty}\frac{n+1}{2n}=\lim_{x\to\infty}\left(\frac{1}{2}+\frac{1}{2n}\right)=\frac{1}{2}$

$\left(\therefore \lim_{x\to\infty}\frac{1}{n}=0\right)$

7. $\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}$

Let $\frac{\pi}{2}-x=t \Rightarrow x=\frac{\pi}{2}-t$

When $x\to\frac{\pi}{2}$, then $t\Rightarrow 0$.

The given limit

$=\lim_{t\to 0}\frac{1-\sin\left(\frac{\pi}{2}-t\right)}{t^2}$

$=\lim_{t\to 0}\frac{1-\cos t}{t^2}$

$=\lim_{t\to 0}\frac{2\sin^2\frac{t}{2}}{t^2}$

$=2\lim_{t\to 0}\frac{\sin^2\frac{t}{2}}{4.\frac{t^2}{4}}$

$=\frac{1}{2}\lim_{t\to 0}\left(\frac{\sin^2\frac{t}{2}}{\frac{t}{2}}\right)^2=\frac{1}{2}.1=\frac{1}{2}$

8. $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin 2x}$

$=\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x.\,2\sin x\cos x}$

$=\frac{1}{2}\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x^2.\frac{\sin x}{x}.\cos x}$

$=\frac{1}{2}.\lim_{x\to 0}\frac{1-\cos x}{x^2}.\lim_{x\to 0}\frac{1}{\left(\frac{\sin x}{x}\right)}$.

$\lim_{x\to 0}\left(\frac{1+\cos x+\cos^2 x}{\cos x}\right)$

$=\frac{1}{2}.\frac{1}{2}.1\ \frac{1+1+1}{1}$

$=\frac{3}{4}$

9. $\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

$=\lim_{x\to 0}\frac{(1-\cos x)+\sin x}{(1-\cos x)-\sin x}$

$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}$

$=\lim_{x\to 0}\frac{2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{2\sin\frac{x}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}$

$=\lim_{x\to 0}\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{x}{2}-\cos\frac{x}{2}}=\frac{0+1}{0-1}=-1$.

10. $\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\tan x$

$\left[\frac{\pi}{2}-x=t\Rightarrow x=\frac{\pi}{2}-t \text{ when } x\to\frac{\pi}{2},\ t\to 0\right]$

$=\lim_{t\to 0} t\tan\left(\frac{\pi}{2}-t\right)$

$=\lim_{t\to 0} t\ \cot\ t$

$=\lim_{t\to 0}\left(\frac{t}{\tan t}\right)=1$

11. $\lim_{t\to 0}\frac{\cos x-\cos 5x}{\cos 2x-\cos 6x}$

$=\lim_{t\to 0}\frac{2\sin\frac{x+5x}{2}\sin\frac{5x-x}{2}}{2\sin\frac{2x+6x}{2}\sin\frac{6x-2x}{2}}$

$=\lim_{t\to 0}\frac{2\ \sin 3x.\sin 2x}{2\sin 4x.\sin 2x}$

$=\lim_{t\to 0}\frac{\sin 3x}{\sin 4x}$

$=\lim_{t\to 0}\frac{\frac{\sin 3x}{x}}{\frac{\sin 4x}{x}}$

$=\frac{\lim_{t\to 0} 3.\left(\frac{\sin 3x}{3x}\right)}{\lim_{t\to 0} 4.\left(\frac{\sin 4x}{4x}\right)}=\frac{3}{4}$.

12. $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}$

$=\lim_{x\to 0}\frac{2\sin x-2\sin x\cos x}{x^3}$

$=\lim_{x\to 0}\frac{2\sin x(1-\cos x)}{x^3}$

$=2-\lim_{x\to 0}\frac{\sin x}{x}.\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$

$=2\ .\ 1\ .\ \frac{1}{2}=1$

13. $\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$

$=\lim_{x\to 0}\frac{(\sqrt{1+\sin x}-\sqrt{1-\sin x})(\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\tan x(\sqrt{1+\sin x}+\sqrt{1-\sin x})}$

$=\lim_{x\to 0}\frac{(1+\sin x)-(1-\sin x)}{\tan x(\sqrt{1+\sin x}+\sqrt{1-\sin x})}$

$=\lim_{x\to 0}\frac{2\sin x}{\frac{\sin x}{\cos x}(\sqrt{1+\sin x}+\sqrt{1-\sin x})}$

$=2.\lim_{x\to 0}\frac{\cos x}{(\sqrt{1+\sin x}+\sqrt{1-\sin x})}$

$=2.\frac{1}{(\sqrt{1+0}+\sqrt{1-0})}=2\text{x}\ \frac{1}{2}=1$.

14. $\lim_{x\to 2}\frac{\log_e(x-1)}{x^2-3x+2}$

$=\lim_{x\to 2}\frac{\log_e(x-1)}{(x-1)(x-2)}$

$=\lim_{x\to 2}.\frac{1}{x-1}\,x\,\lim_{x\to 2}\frac{\log_e(x-1)}{x-2}$

$=1\,x\,\lim_{x\to 2}.\frac{\log_e(x-1)}{x-2}$

$=1.\lim_{t\to 0}\frac{\log_e(2+t-1)}{t}$

$(x - 2 = t, x = 2 + t \text{ when } x \to 2,\ t \to 0\ )$

$=\lim_{t\to 0}\frac{\log_e(1+t)}{t}=1$

15. $\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$

$=\lim_{x\to 5}\frac{(\sqrt{x-1}-2)(\sqrt{x-1}+2)}{(x-5)(\sqrt{x-1}+2)}$

$=\lim_{x\to 5}\frac{(x-1-4)}{(x-5)(\sqrt{x-1}+2)}$

$=\lim_{x\to 5}\frac{(x-5)}{(x-5)(\sqrt{x-1}+2)}$

$=\lim_{x\to 5}\frac{1}{\sqrt{x-1}+2}=\frac{1}{2+2}=\frac{1}{4}.$

16. $\lim_{x\to 0}\frac{3^x-2^x}{4^x-3^x}$

$=\lim_{x\to 0}\frac{(3^x-2^x)/x}{(4^x-3^x)/x}$

$=\frac{\lim_{x\to 0}\left(\frac{3^x-2^x}{x}\right)}{\lim_{x\to 0}\left(\frac{4^x-3^x}{x}\right)}$

$=\frac{\lim_{x\to 0}\left(\frac{3^x-1-2^x+1}{x}\right)}{\lim_{x\to 0}\frac{4^x-1-3^x+1}{x}}$

$=\frac{\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)-\left(\frac{2^x-1}{x}\right)}{\lim_{x\to 0}\left(\frac{4^x-1}{x}\right)-\left(\frac{3^x-1}{x}\right)}$

$=\frac{\ln 3-\ln 2}{\ln 4-\ln 3}.$

17. $\lim_{x\to 1}\frac{2^{x-1}-1}{\sqrt{x}-1}$

$=\lim_{x\to 1}\frac{(2^{x-1}-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}$

$=\lim_{x\to 1}\frac{(2^{x-1}-1)(\sqrt{x}+1)}{x-1}$

$=\lim_{x\to 1}\frac{2^{x-1}-1}{x-1}.\lim_{x\to 1}(\sqrt{x}+1)$

$=\ln 2\,x\,2=2\ln 2.$

18. $\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}}{x}$

$=\lim_{x\to 0}\frac{(a-\sqrt{a^2-x^2})(a+\sqrt{a^2-x^2})}{x(a+\sqrt{a^2-x^2})}$

$=\lim_{x\to 0}\frac{a^2-(a^2-x^2)}{x(a+\sqrt{a^2-x^2})}$

$=\lim_{x\to 0}\frac{x^2}{x(a+\sqrt{a^2-x^2})}$

$=\lim_{x\to 0}\frac{x}{a^2+\sqrt{a^2-x^2}}=0$

19. $\lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}}$

$= \lim_{\theta-0} \frac{\tan 2\left(\frac{\pi}{2}+\theta\right)}{\theta}$

$[x-\frac{\pi}{2}=\theta \;\Rightarrow x=\frac{\pi}{2}+\theta \; x\to\frac{\pi}{2} \Rightarrow \theta\to 0)$

$= \lim_{\theta\to 0} \frac{\tan(\pi+2\theta)}{\theta}$

$= \lim_{\theta\to 0} \frac{\tan 2\theta}{\theta}$

$= 2.\lim_{\theta\to 0}\left(\frac{\tan 2\theta}{2\theta}\right)$

$= 2 \text{ x } 1 = 2.$

20. $\lim_{\theta\to 0} \frac{\sin ax + bx}{ax+\sin bx}$

$= \lim_{\theta\to 0} \frac{\frac{\sin ax+bx}{x}}{\frac{ax+\sin bx}{x}}$

$= \frac{\lim_{\theta\to 0} \frac{\sin ax}{x}+b}{\lim_{\theta\to 0} a+\frac{\sin bx}{x}}$

$= \frac{a+b}{a+b} = 1.$

21. $\lim_{x\to b} \frac{\sqrt{x-a}-\sqrt{b-a}}{x^2-b^2}$

$= \lim_{x\to b} \frac{\left(\sqrt{x-a}-\sqrt{b-a}\right)\left(\sqrt{x-a}+\sqrt{b-a}\right)}{\left(x^2-b^2\right)\left(\sqrt{x-a}+\sqrt{b-a}\right)}$

$= \lim_{x\to b} \frac{(x-a)-(b-a)}{(x-b)(x+b)\left(\sqrt{x-a}+\sqrt{b-a}\right)}$

$= \lim_{x\to b} \frac{x-b}{(x-b)(x+b)\left(\sqrt{x-a}+\sqrt{b-a}\right)}$

$= \lim_{x\to b} \frac{1}{(x+b)\left(\sqrt{x-a}+\sqrt{b-a}\right)}$

$= \frac{1}{2b\left(\sqrt{b-a}+\sqrt{b-a}\right)} = \frac{1}{4b\sqrt{b-a}}$

22. $\lim_{x\to b} \frac{\sqrt{x-a}-\sqrt{b-a}}{x^2-b^2}$

$\lim_{x\to b} \frac{(\sqrt{x-a}-\sqrt{b-a})(\sqrt{x-a}+\sqrt{b-a})}{(x^2-b^2)(\sqrt{x-a}+\sqrt{b-a})}$

$\lim_{x\to b} \frac{(x-a)-(b-a)}{(x-b)(x+b)(\sqrt{x-a}+\sqrt{b-a})}$

$\lim_{x\to b} \frac{x-b}{(x-b)(x+b)(\sqrt{x-a}+\sqrt{b-a})}$

$\lim_{x\to b} \frac{1}{(x+b)(\sqrt{x-a}+\sqrt{b-a})}$

$= \frac{1}{2b(\sqrt{b-a}+\sqrt{b-a})} = \frac{1}{4b\sqrt{b-a}}.$

23. Let $y = \sec x$

$\therefore \frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{\sec(x+\Delta x)-\sec x}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{\frac{1}{\cos(x+\Delta x)}-\frac{1}{\cos x}}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{\cos x-\cos(x+\Delta x)}{\cos(x+\Delta x).\cos x.\Delta x}$

$= \lim_{\Delta x\to 0} \frac{2\sin\frac{x+x+\Delta x}{2}.\sin\frac{x+\Delta x-x}{2}}{\cos(x+\Delta x).\cos x.\Delta x}$

$= \lim_{\Delta x\to 0} \frac{2\sin\left(x+\frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}}{\cos(x+\Delta x)\,\cos x\,\Delta x}$

$= \lim_{\Delta x\to 0} \frac{\sin\left(x+\frac{\Delta x}{2}\right)}{\cos(x+\Delta x)\,\cos x} \lim_{\Delta x\to 0}\left(\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}\right)$

$= \frac{\sin x}{\cos x.\cos x}.1 = \frac{1}{\cos x}.\frac{\sin x}{\cos x}$

$= \sec x.\tan x.$

24. Let $y=\frac{1}{x^2}$

$$\therefore \frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\frac{1}{(x+\Delta x)^2}-\frac{1}{x^2}}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{x^2-(x+\Delta x)^2}{x^2(x+\Delta x)^2\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{(x+x+\Delta x)(x-x-\Delta x)}{x^2(x+\Delta x)^2.\,\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{(2x+\Delta x)(-\Delta x)}{x^2(x+\Delta x)^2.\Delta x}$$

$$=-\lim_{\Delta x\to 0}\frac{2x+\Delta x}{x^2(x+\Delta x)^2}$$

$$=-\frac{2x}{x^2.\,x^2}=-\frac{2}{x^3}.$$

25. Let $y=x\sin x$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{(x+\Delta x)\sin(x+\Delta x)-x\sin x}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{x[\sin(x+\Delta x)-\sin x]+\Delta x\sin(x+\Delta x)}{\Delta x}$$

$$=x.\lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}+\lim_{\Delta x\to 0}\sin(x+\Delta x)$$

$$=x.\lim_{\Delta x\to 0}\frac{2\cos\left(\frac{x+\Delta x+x}{2}\right)\sin\left(\frac{x+\Delta x-x}{2}\right)}{\Delta x}+\sin x$$

$$=x.\lim_{\Delta x\to 0}\cos\left(\frac{2x+\Delta x}{2}\right).\left(\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}\right)+\sin x$$

$$=x\cos x+\sin x\ .$$

26. We know

$$|x|=\begin{cases}x & \text{when } x\ge 0\\ -x & \text{when} < 0\end{cases}$$

$$|x-1|=\begin{cases}x-1 & \text{when } x\ge 1\\ -(x-1) & \text{when} < 1\end{cases}$$

When $x<0$, $f(x)=|x|+|x-1|$

$$=-x-(x-1)=-2x+1$$

When $0\le x<1$, $f(x)=|x|+|x-1|$

$$=x-(x-1)=1$$

When $x\ge 1, f(x)=|x|+|x-1|$

$$=x+x-1=2x-1$$

When $x<1$, $\frac{df(x)}{dx}=d\frac{(-2x+1)}{dx}=-2$

When $0\le x<1$, $\frac{df(x)}{dx}=\frac{d1}{dx}=0$

When $x\ge 1$, $\frac{df(x)}{dx}=\frac{d(2x-1)}{dx}=2$.

27. $y=\frac{x+\sin x}{1+\cos x}$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x+\sin x}{1+\cos x}\right)$$

$$=\frac{(1+\cos x)\frac{d}{dx}(x+\sin x)-(x+\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$=\frac{(1+\cos x)(1+\cos x)-(x+\sin x).(-\sin x)}{(1+\cos x)^2}$$

$$=\frac{1+2\cos x+\cos^2 x+x\sin x+\sin^2 x}{(1+\cos x)^2}$$

$$=\frac{2+2\cos x+x\sin x}{(1+\cos x)^2}.$$

28. $y=x^2\cos x$

$$\frac{dy}{dx}=\frac{d(x^2\cos x)}{dx}$$

$$=x^2.\frac{d\cos x}{dx}+\cos x.\frac{dx^2}{dx}$$

$$=-x^2\sin x+2x\cos x\cdot$$

29. $y = \frac{x^n}{\tan x}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^n}{\tan x}\right)$

$= \frac{\tan x . \frac{dx^n}{dx} - x^n . \frac{d \tan x}{dx}}{\tan^2 x}$

$= \frac{nx^{n-1} \tan x - x^n \sec^2 x}{\tan^2 x}.$

30. $y = \frac{1 - \tan x}{1 + \tan x}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1 - \tan x}{1 + \tan x}\right)$

$= \frac{(1 + \tan x)\frac{d}{dx}(1 - \tan x) - (1 - \tan x).\frac{d}{dx}(1 + \tan x)}{(1 + \tan x)^2}$

$= \frac{(1 + \tan x)(-\sec^2 x) - (1 - \tan x).\sec^2 x}{(1 + \tan x)^2}$

$= \frac{-\sec^2 x - \tan x \sec^2 x + \sec^2 x - \tan x \sec^2 x}{(1 + \tan x)^2}$

$= \frac{-2 \sec^2 x}{(1 + \tan x)^2}$

31. $y = \frac{\cos x}{1 + x^2}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\cos x}{1 + x^2}\right)$

$= \frac{(1 + x^2)\frac{d \cos x}{dx} - \cos x \frac{d(1 + x^2)}{dx}}{(1 + x^2)^2}$

$= \frac{(1 + x^2)(-\sin x) - 2x \cos x}{(1 + x^2)^2}$

$= \frac{-\sin x - x^2 \sin x - 2x \cos x}{(1 + x^2)^2}.$

32. $y = \tan^2 x + a^x$

$\frac{dy}{dx} = \frac{d(\tan^2 x + a^x)}{dx}$

$= \frac{d \tan^2 x}{dx} + \frac{da^x}{dx}$

$= \frac{d \tan^2 x}{d \tan x} . \frac{d \tan x}{dx} + a^x \ln a$

$= 2 \tan x \sec^2 x + a^x \ln a$

33. $y = \frac{e^x + e^{-x}}{x^2 + 1}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{x^2 + 1}\right)$

$= \frac{(x^2 + 1)d\frac{(e^x + e^{-x})}{dx} - (e^x + e^{-x}).\frac{d(x^2 + 1)}{dx}}{(x^2 + 1)^2}$

$= \frac{(x^2 + 1)(e^x - e^{-x}) - 2x(e^x + e^{-x})}{(x^2 + 1)^2}.$

34. $y = \frac{2x + 1}{x^2 + 1}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x + 1}{x^2 + 1}\right)$

$= \frac{(x^2 + 1)d\frac{(2x + 1)}{dx} - (2x + 1)\frac{d(x^2 + 1)}{dx}}{(x^2 + 1)^2}$

$= \frac{(x^2 + 1).2 - (2x + 1).2x}{(x^2 + 1)^2}$

$= \frac{2x^2 + 2 - 4x^2 - 2x}{(x^2 + 1)^2}$

$= \frac{2 - 2x - 2x^2}{(x^2 + 1)^2}.$

35. $y = x^3.\sin x.e^{4\ln x}$

$= x^3.\sin x.e^{\ln x^4}$

$= x^3 \sin x.x^4$

$= x^7 \sin x$

$\frac{dy}{dx} = \frac{d(x^7 \sin x)}{dx}$

$= x^7.\frac{d\sin x}{dx} + \sin x.\frac{dx^7}{dx}$

$= x^7.\cos x + 7x^6 \sin x$

36. $y = \tan^2 x + \sec^2 x$

$= \tan^2 x + 1 + \tan^2 x$

$= 2\tan^2 x + 1$

$\frac{dy}{dx} = \frac{d(2\tan^2 x + 1)}{dx}$

$= 2.\frac{d\tan^2 x}{dx} + \frac{d1}{dx}$

$= 2.\frac{d\tan^2 x}{d\tan x}.\frac{d\tan x}{dx} + 0$

$= 2.2\tan x.\sec^2 x$

$= 4\tan x.\sec^2 x$.

37. $y = \cos^2 x + e^x \cos x$

$\frac{dy}{dx} = \frac{d(\cos^2 x + e^x \cos x)}{dx}$

$= \frac{d\cos^2 x}{dx} + \frac{d(e^x \cos x)}{dx}$

$= \frac{d\cos^2 x}{d\cos x}.\frac{d\cos x}{dx} + e^x.\frac{d(e^x \cos x)}{dx}$

$= \frac{d\cos^2 x}{d\cos x}.\frac{d\cos x}{dx} + e^x.\frac{d\cos x}{dx} + \cos x\frac{de^x}{dx}$

$= 2\cos x.(-\sin x) + e^x(-\sin x) + \cos x.e^x$

$= -\sin 2x - e^x \sin x + e^x \cos x \cdot$

38. $y = ax^2 + b\tan x + \ln x^3$

$= ax^2 + b\tan x + 3\ln x$

$\frac{dy}{dx} = \frac{d(ax^2 + b\tan x + 3\ln x)}{dx}$

$= a.\frac{dx^2}{dx} + b.\frac{d\tan x}{dx} + 3.\frac{d\ln x}{dx}$

$= 2ax + b\sec^2 x + \frac{3}{x}$.

39. $y = \frac{\sec^2 x}{\tan x + \cos x} = \frac{\sec^2 x}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$

$= \frac{\sec^2 x}{\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)}$

$= \frac{\sec^2 x}{\left(\frac{1}{\sin x \cos x}\right)} = \frac{1}{\cos^2 x}.\sin x \cos x$

$= \frac{\sin x}{\cos x} = \tan x$

$\frac{dy}{dx} = \frac{d\tan x}{dx} = \sec^2 x$.

40. $y = \frac{\cos 2x}{\tan x - \cot x}$

$= \frac{\cos 2x}{\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right)}$

$= \frac{\cos 2x.\sin x \cos x}{\sin^2 x - \cos^2 x}$

$= -\frac{\cos 2x.\sin x \cos x}{\cos^2 x - \sin^2 x}$

$$=-\frac{\cos 2x.\sin x\cos x}{\cos 2x}$$

$$=-\frac{1}{2}.2\sin x\cos x=-\frac{1}{2}\sin 2x$$

$$\frac{dy}{dx}=-\frac{1}{2}.\frac{d\sin 2x}{dx}$$

$$=-\frac{1}{2}.2\cos 2x=-\cos 2x.$$

41. $y=\dfrac{\cos x}{1+\sin x}$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\cos x}{1+\sin x}\right)$$

$$=\frac{(1+\sin x).\frac{d\cos x}{dx}-\cos x.\frac{d(1+\sin x)}{dx}}{(1+\sin x)^2}$$

$$=\frac{(1+\sin x)(-\sin x)-\cos x.\cos x}{(1+\sin x)^2}$$

$$=\frac{-\sin x-(\sin^2 x+\cos^2 x)}{(1+\sin x)^2}$$

$$=\frac{-\sin x-1}{(1+\sin x)^2}=\frac{-1}{1+\sin x}.$$

42. $y=\sqrt{1+\sin 2x}$

$$=\sqrt{\cos^2 x+\sin^2 x+2\sin x\cos x}$$

$$=\sqrt{(\cos x+\sin x)^2}$$

$$=\cos x+\sin x$$

$$\frac{dy}{dx}=\frac{d(\cos x+\sin x)}{dx}$$

$$=\frac{d\cos x}{dx}+\frac{d\sin x}{dx}$$

$$=-\sin x+\cos x\cdot$$

43. $y=\sqrt{1-\cos 2x}=\sqrt{2\sin^2 x}$

$$=\sqrt{2}\sin x$$

$$\frac{dy}{dx}=\sqrt{2}\frac{d\sin x}{dx}=\sqrt{2}\cos x.$$

44. $y=\sqrt{\left(\dfrac{\sec x+\tan x}{\sec x-\tan x}\right)}$

$$=\sqrt{\frac{(\sec x+\tan x)(\sec x+\tan x)}{(\sec x-\tan x)(\sec x+\tan x)}}$$

$$=\sqrt{\frac{(\sec x+\tan x)^2}{\sec^2 x-\tan^2 x)}}=\sec x+\tan x$$

$$\frac{dy}{dx}=\frac{d(\sec x+\tan x)}{dx}$$

$$=\frac{\sec x}{dx}+\frac{d\tan x}{dx}$$

$$=\sec x\tan x+\sec^2 x\cdot$$

45. Given that

$$f(x)=\begin{cases} x\sin\frac{1}{x} & \text{when } x\neq 0\\ 0 & \text{when } x=0\end{cases}$$

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{x\to 0}\frac{(x)\sin\frac{1}{x}-0}{x}$$

$=\lim\limits_{x\to 0}\sin\dfrac{1}{x}$ which lies between -1 and 1 and hence does not exist.

So that function is not differentiable at x= 0.

6. Mathematical Reasoning, Statistics and Probability

1. Given statement is

$\cos x = 0 \rightarrow x = \frac{\pi}{2}$.

Its contrapositive is $x \neq \frac{\pi}{2} \rightarrow \cos x \neq 0$.

Its converse is $x = \frac{\pi}{2} \rightarrow \cos x = 0$.

2. The given statement is

$2x + 3 = 9 \rightarrow x \neq 5$.

Its contrapositive is

$x = 5 \rightarrow 2x + 3 \neq 9$

Its converse is $x \neq 5 \rightarrow 2x + 3 = 9$.

3.(i) The given statement is 30 is divisible by 2, 3 and 5.

The component statements are

p : 30 is divisible by 2

q : 30 is divisible by 3

r : 30 is divisible by 5

Above three statements are true.

The connecting word is 'and'.

(ii) The given statement is

"All rational or irrational numbers are real numbers.

The component statements are

p : All rational numbers are real numbers

q : All irrational numbers are real numbers

Here both the statements are true and the connecting word is 'or'.

(iii) The given statement is

"The constituents of water are oxygen and nitrogen.

The component statements are

p : The constituent of what is oxygen.

q : The constituent of water is nitrogent.

The 1st statement is true and 2nd statement is false.

The connecting word is 'and'.

4. The given statement is

"For any real number a and is, $a^2 = b^2$ implies that a = b."

Let a = 2, b = -2

$2^2 = (-2)^2$

But $2 \neq -2$

So the given statement is not true.

5. The given statement is

"The equation $x^2 - 1 = 0$ doesnot have a root lying between 0 and 2".

The given equation $x^2 - 1 = 0$.

x = 1 is a root of the equation.

Here x = 1 lies between 0 and 2.

$\rightarrow$ The given statement is not true.

6. The given statement is

"If x is an integer and x^2 is odd then x is odd.

Let p : x is an integer and x^2 is odd

q : x is odd.

We shall prove the given statement by the method of contrapositive.

We shall show that $\sim q \rightarrow \sim p$.

Let x is not odd

$\Rightarrow$ x is even

$\Rightarrow x = 2k$ were $k \in Z$

$\Rightarrow x^2 = (2k)^2$

$\Rightarrow x^2 = 4k^2$

$\Rightarrow x^2$ is even

$\Rightarrow x^2$ is not odd

So $\sim q \rightarrow \sim p$

i.e. if x is not odd then x^2 is not odd.

$\Rightarrow$ If x is an integer and x^2 and odd then x is odd.

7.(i) The given statement is x is an even number implies that x is divisible by 4.

Let p : x is an even number

q : x is divisible by 4.

The given statement is $p \rightarrow q$.

Its contrapositive is $\sim q \rightarrow \sim p$.

i.e. if x is not divisible by 4 then x is not an even number.

Its converse is $q \rightarrow p$

i.e. if x is divisible by 4 then x is an even number.

(ii) The given statement is

"If a number n is even then n^2 is even.

Let p : Number n is even.

q : n^2 is even.

The given statement is $p \rightarrow q$.

Its contrapositive is $\sim q \rightarrow \sim p$.

i.e. if n^2 is not even then n is not even.

Its inverse is $q \rightarrow p$.

i.e. if n^2 is even then n is even.

8.

1	2	3	4	5
p	q	$\sim q$	$\sim q \vee q$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

The column (1) and (5) have the same truth values.

$\therefore p \wedge (\sim q \vee q) \equiv p$.

9.

1	2	3	4	5	6
p	q	$\sim p$	$\sim q$	$\sim p \leftrightarrow q$	$p \leftrightarrow \sim q$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

The column (5) and (6) have the same truth values.

$\therefore \sim p \leftrightarrow q \equiv p \leftrightarrow \sim q$.

10.

1	2	3	4	5	6
p	q	$\sim p$	$p \leftrightarrow q$	$\sim (p \leftrightarrow q)$	$\sim p \leftrightarrow q$
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	F

The column (5) and (6) have the same truth values.

So $\sim (p \leftrightarrow q) \equiv \sim p \leftrightarrow q$.

11.

1	2	3	4
p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
F	F	T	T
F	T	T	F
F	F	F	F

Here we see that the columns (1) and (4) have the same truth values.

$\therefore$ So the two statements are equivalent.

i.e. $p \wedge (p \vee q) \equiv p$.

12.

p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(\sim p \wedge q) \rightarrow p \vee q$
T	T	F	F	T	T
T	F	F	F	T	T

The truth value of $(\sim p \wedge q) \rightarrow (p \vee q)$ is always true.

13.

p	q	$\sim p$	$p \vee q$	$\sim p \wedge q$	$p \vee q \to \sim p \wedge q$
F	T	T	T	T	T
F	F	T	F	F	T

The truth value of $p \vee q \to \sim p \wedge q$ is always true.

14. The given statement is

"If x is an integer and x^2 is even then x is also even".

Let p : x is an integer and x^2 is even

q : x is an even integer.

The given statement $p \to q$.

We shall show that $\sim q \to \sim p$.

$\sim q$: x is not an even integer

$\Rightarrow$ x is an odd integer

$\Rightarrow x = 2n + 1$, where $n \in Z$.

$\Rightarrow x^2 = (2n+1)^2$

$= 4n^2 + 4n + 1$

$= 2n(2n+2) + 1$

$\Rightarrow x^2$ is an odd integer

$\Rightarrow \sim p$

$\therefore \sim p \Rightarrow \sim p$ which is contrapositive

so $p \Rightarrow q$ is true.

15. The given statement is

"If x is real number such that $x^3 + 4x = 0$ then $x = 0$.

Here x is real number such that $x^3 + 4x = 0$

$\Rightarrow$ x is real number such that $x(x^2+4) = 0$.

$\Rightarrow x = 0 \quad (\because x \in R,\ x^2 + 4 \neq 0)$.

16. Range = Maximum value - Minimum value. This gives the extent up to which the data fluctuates.

Range $= 104^0 F - 99^0 F = 5^0 F$.

17. The data is 30, 40, 85, 75, 45.

Let $\bar{x}$ be the arithmetic mean.

$$\bar{x} = \frac{30+40+85+75+45}{5} = \frac{275}{5} = 55$$

We find the mean deviation from the following table.

x_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$
30	−25	25
40	−15	15
85	30	30
75	20	20
45	−10	10

$\therefore \sum \lvert x_i - \bar{x} \rvert = 100$

$$\text{M.D.} = \frac{\sum \lvert x_i - \bar{x} \rvert}{5} = \frac{100}{5} = 20.$$

18. The given data is tabulated as shown below.

Age (x_1)	Frequency (f_1)	$f_1 x_1$	$\lvert x_i - \bar{x} \rvert$	$f_r \lvert x_i - \bar{x} \rvert$
14	5	70	2	10
15	4	60	1	4
30	1	30	14	14

$\sum f_i = 10, \quad \sum f_i x_i = 160$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{160}{10} = 16$$

$$\text{M.D.} = \frac{\sum f_i \lvert x_i - \bar{x} \rvert}{\sum f_i} = \frac{28}{10} = 2.8.$$

19.(i) For a simple distribution if $x_1, x_2, \ldots x_n$ be the n variables and $\bar{x}$ be their arithmatic mean, then the mean deviation is defined as

$$\text{M.D.} = \frac{\lvert x_1 - \bar{x} \rvert + \lvert x_2 - \bar{x} \rvert + \ldots + \lvert x_n - \bar{x} \rvert}{n}$$

$$= \frac{\sum_{i=1}^{n} \lvert x_i - \bar{x} \rvert}{n}.$$

(ii) In a frequency distribution if $x_1, x_2, ...x_n$ be the variables with the corresponding frequency $f_1, f_2, ... f_n$ and $\bar{x}$ be the arithmetic mean, then the mean deviation is defined as

$$\text{M.D.} = \frac{f_1|x_1 - \bar{x}| + f_2|x_2 - \bar{x}| + ... + f_n|x_n - \bar{x}|}{f_1 + f_2 + ... + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i|x_i - \bar{x}|}{\sum_{i=1}^{n} f_i}.$$

(iii) For a grouped frequency distribution if $x_1, x_2, ..., x_n$ be the mid values with the corresponding freqencies $f_1, f_2, ..., f_n$ and $\bar{x}$ be their arithmetic mean, then mean deviation is defined as

$$\text{M.D.} = \frac{f_1|x_1 - \bar{x}| + f_2|x_2 - \bar{x}| + ... + f_n|x_n - \bar{x}|}{f_1 + f_2 + ... + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i|x_i - \bar{x}|}{\sum_{i=1}^{n} f_i}.$$

20. We know the standard deviation σ is given by

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}$$

$$= \sum_{i=1}^{n} \frac{f_i(x_i^2 + \bar{x}^2 - 2x_i\bar{x})}{\sum_{i=1}^{n} f_i}$$

$$= \frac{\sum f_i x_i^2 + \sum f_i \bar{x}^2 - 2\bar{x}\sum f_i x_i}{\sum f_i}$$

$$= \frac{\sum f_i x_i^2 + \bar{x}^2 \sum f_i - 2\bar{x}\sum f_i x_i}{\sum f_i}$$

$$= \frac{\sum f_i x_i^2}{\sum f_i} + \bar{x}^2 - 2\bar{x}.\frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{\sum f_i x_i^2}{\sum f_i} + \bar{x}^2 - 2\bar{x}.\bar{x}$$

$$= \frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2$$

$$= \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

21. The coefficient of variation is defined as

$\text{C.V.} = \frac{\sigma}{\bar{x}} \text{x} 100, \ \bar{x} \neq 0$. Where σ is the standard deviation and $\bar{x}$ is the mean of the data.

Here $x_1 = 15, \ x_2 = 20, \ x_3 = 25, \ x_4 = 30, \ x_5 = 35$.

$$\text{Mean } \bar{x} = \frac{\sum x_i}{5} = \frac{15 + 20 + 25 + 30 + 35}{5}$$

$$= \frac{125}{5} = 25.$$

To find the standard deviation, we form the following table.

Wage x_i	$x_i - \bar{x}$	$(x - \bar{x})^2$
15	−10	100
20	−5	25
25	0	0
30	5	25
35	10	100
		250

Standard deviation

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{5}} = \sqrt{\frac{250}{5}} = \sqrt{50} = 5\sqrt{2}$$

Coefficient of variation

$$= \frac{\sigma}{x} \text{ x } 100 = \frac{5\sqrt{2}}{25} \text{ x} 100 = 20\sqrt{2}.$$

22. First n natural numbers are 1, 2, 3, ..., n.

To find the standard deviation, we form the following table.

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
1	1	1	1^2	1^2
2	1	2	2^2	2^2
3	1	3	3^2	3^2
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
n	1	n	n^2	n^2

$\sum f_i = n$

$\sum f_i x_i = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

$\sum f_i x_i^2 = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

$$(S.D.)^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{n^2 - 1}{12}$$

$$\Rightarrow S.D. = \sqrt{\frac{n^2 - 1}{12}}$$

23. The given data is 1, 2, 3, 5.

Mean $\bar{x} = \frac{1+2+3+5}{4} = \frac{11}{4}$

Mean deviation

$$= \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + |x_3 - \bar{x}| + |x_4 - \bar{x}|}{4}$$

$$= \frac{\left|1 - \frac{11}{4}\right| + \left|2 - \frac{11}{4}\right| + \left|3 - \frac{11}{4}\right| + \left|5 - \frac{11}{4}\right|}{4}$$

$$= \frac{1}{4}\left(\frac{7}{4} + \frac{3}{4} + \frac{1}{4} + \frac{9}{4}\right) = \frac{20}{4.4} = \frac{5}{4}.$$

24. To find the standard deviation, we form the following table.

x_i	f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
1	1	1	1	1
2	1	4	2	4
3	1	9	3	9
5	1	25	5	25

$\sum f_i x_i = 1 + 2 + 3 + 5 = 11$

$\sum f_i x_i^2 = 1 + 4 + 9 + 25 = 39$

$$(S.D.)^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

$$= \frac{39}{4} - \left(\frac{11}{4}\right)^2$$

$$= \frac{39}{4} - \frac{121}{16}$$

$$= \frac{156 - 121}{16} - \frac{35}{16}$$

$$\Rightarrow S.D. = \frac{\sqrt{35}}{4}.$$

25. We form the following table to get the standard deviation.

x_i	f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
1	1	1^2	1^2	1^3
2	2	2^2	2^2	2^3
3	3	3^2	3^2	3^3
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
n	n	n^2	n^2	n^3

Here $\sum f_i = 1+2+3+...+n = \frac{n(n+1)}{2}$

$\sum f_i x_i = 1^2+2^2+3^2+...+n^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum f_i x_i^2 = 1^3+2^3+3^3+...+n^3 = \frac{n^2(n+1)^2}{4}$

Standard deviation is given by

$$(S.D.)^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

$$= \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2}} - \left(\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}\right)^2$$

$$= \frac{n(n+1)}{2} - \left(\frac{2n+1}{3}\right)^2$$

$$= \frac{n(n+1)}{2} - \frac{(2n+1)^2}{9}$$

$$= \frac{9n(n+1)-2(2n+1)^2}{18}$$

$$= \frac{9n^2+9n-8n^2-8n-2}{18}$$

$$= \frac{n^2+n-2}{18}$$

$$\therefore S.D. = \sqrt{\frac{n^2+n-2}{18}}$$

26. To get the standard deviation, we form the following table

x_i	f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
1	1	1^2	1	1^2
3	1	3^2	3	3^2
5	1	5^2	5	5^2
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$(2n-1)$	1	$(2n-1)^2$	$2n-1$	$(2n-1)^2$

$\sum f_i = n$

$\sum f_i x_i = 1+3+5+...+(2n-1) = n^2$

$\sum f_i x_i^2 = 1^2+3^2+5^2+...+(2n-1)^2$

$= \frac{1}{3}n(4n^2-1)$

The standard deviation is given by

$$(S.D.)^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

$$= \frac{\frac{1}{3}n(4n^2-1)}{n} - \left(\frac{n^2}{n}\right)^2$$

$$= \frac{1}{3}(4n^2-1) - n^2$$

$$= \frac{4n^2-1-3n^2}{3} = \frac{n^2-1}{3}$$

$$\Rightarrow S.D. = \sqrt{\frac{n^2-1}{3}}.$$

27. The given data is 5, 15, 20, 30, 40.

Mean $\bar{x} = \frac{5+15+20+30+40}{5}$

$= \frac{110}{5} = 22.$

To get the mean deviation, we form the following table.

x_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x}\rvert$
5	−17	17
15	−7	7
20	−2	2
30	8	8
40	18	18

$$\Sigma\lvert x_1 - \bar{x}\rvert$$

Mean deviation is given by

$$\text{M.D.} = \frac{\Sigma\lvert x_i - \bar{x}\rvert}{5} = \frac{52}{5}.$$

28. To get the mean and variance, we form the following table.

x_i	f_i	$x_1{}^2$	f_ix_i	$f_ix_1{}^2$
2	1	2^2	2	2^2
4	1	4^2	4	4^2
6	1	6^2	6	6^2
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
2n	1	$(2n)^2$	2n	$(2n)^2$

$$\Sigma f_i = n$$

$$\Sigma f_ix_i = 2+4+6+\ldots+2n = n(n+1)$$

$$\Sigma f_ix_i = 2^2+4^2+6^2+\ldots+2n^2$$

$$= 4.n\frac{(n+1)(2n+1)}{6} = \frac{2}{3}n(n+1)(2n+1)$$

$$\text{Mean } \bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{n(n+1)}{n} = n+1$$

$$\text{Variance} = \frac{\Sigma f_ix_i{}^2}{\Sigma f_i} - \left(\frac{\Sigma f_ix_i}{\Sigma f_i}\right)^2$$

$$= \frac{2}{3}\frac{n(n+1)(2n+1)}{n} - (n+1)^2$$

$$= \frac{2}{3}(n+1)(2n+1) - (n+1)^2$$

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{3}$$

$$= \frac{4n^2+6n+2-3n^2-6n-3}{3}$$

$$= \frac{n^2-1}{3}.$$

29. The given data is 7, 17, 20, 22, 24.

$$\text{Mean } \bar{x} = \frac{7+17+20+22+24}{5} = \frac{90}{5} = 18$$

To the get the mean deviation, we form the following table.

x_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x}\rvert$
7	−11	11
17	−1	1
20	2	2
22	4	4
24	6	6

$$\Sigma\lvert x_1 - \bar{x}\rvert = 24$$

$$\text{M.D.} = \frac{\Sigma\lvert x_i - \bar{x}\rvert}{5} = \frac{24}{5}.$$

30. Refer No. 28.

31. Two events A and B are mutually exclusive. Let S be the sample space.

$$\therefore\ A \cap B = \varphi$$

$$\therefore \lvert A\cup B\rvert = \lvert A\rvert + \lvert B\rvert$$

$$\frac{\lvert A\cup B\rvert}{\lvert S\rvert} = \frac{\lvert A\rvert}{\lvert S\rvert} + \frac{\lvert B\rvert}{\lvert S\rvert}$$

$$\Rightarrow P(A\cup B) = P(A) + P(B)$$

32. Here A and B are two events.

$$\therefore \lvert A\cup B\rvert = \lvert A\rvert + \lvert B\rvert - \lvert A\cap B\rvert$$

Dividing both sides by $\lvert S\rvert$, we get

$$\frac{\lvert A\cup B\rvert}{\lvert S\rvert} = \frac{\lvert A\rvert}{\lvert S\rvert} + \frac{\lvert B\rvert}{\lvert S\rvert} - \frac{\lvert A\cap B\rvert}{\lvert S\rvert}$$

$$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B).$$

33. Here A and B are two events and $A \subset B$.

$\therefore$ A and B - A are disjoint and their union is B.

$\therefore B = A \cup (B - A)$

$P(B) = P[A \cup (B - A)]$

$= P(A) + P(B - A)$

$\Rightarrow P(B - A) = P(B) - P(A)$

Since $P(B - A) \geq 0$

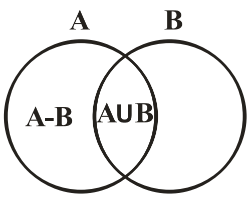
$\Rightarrow P(B) - P(A) \geq 0$

$\Rightarrow P(B) \geq P(A)$

$\Rightarrow P(A) \leq P(B)$

34. Here A and B are two events.

We see that A - B and $A \cap B$ are two mutually exclusive events and their union is A.



$\therefore (A - B) \cup (A \cap B) = A$

$\Rightarrow P[(A - B) \cup (A \cap B)] = P(A)$

$\Rightarrow P(A - B) + P(A \cap B)] = P(A)$

$\Rightarrow P(A - B) = P(A) - P(A \cap B)$.

35. A and A^C are mutually exclusive and their union is S.

$\therefore A \cup A^C = S$

$\Rightarrow P(A \cup A^C) = P(S)$

$\Rightarrow P(A) + P(A^C) = 1$

$\Rightarrow P(A^C) = 1 - P(A)$.

36. A die is thrown twice.
Let S be the sample space
$|S| = 36$.
Let E be the event that the sum of points obtained is 8.

$E = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$\therefore |E| = 5$

$P(E) = \frac{|E|}{|S|} = \frac{5}{36}$.

37. A die is thrown 5 times.

Let S be the sample space.

$|S| = 6^5$

Let A be the event of setting exactly 3 fours.

There are 5 fours.

3 fours can be takedn out of 5 fours in C(5,3) ways. Other two outcomes are taken in 5 x 5 = 5^2 ways.

Total number of outcomes = C(5, 3) x 5^2.

$P(A) = \frac{|A|}{|S|} = \frac{C(5,3) \times 5^2}{6^5}$

$= \frac{5 . 4 . 3}{3 . 2 . 1 \times 6^5} \times 25 = \frac{250}{6^5}$.

38. Here A and B are two events and S be the sample space.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

We know $P(A \cup B) \leq 1$

$\Rightarrow -P(A \cup B) \leq -1$

$\Rightarrow P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1$

$\Rightarrow P(A \cap B) \geq P(A) + P(B) - 1$.

39. There are 100 cards which are numbered 1 to 100. Let $|S|$ be the sample space.

$\therefore |S| = 100$.

Let E be the event of getting card whose number is divisible by 5.

E = {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100}.

$|E| = 20$

$P(E) = \frac{|E|}{|S|} = \frac{20}{100} = \frac{1}{5}$

40. Two cards are drawn form a pack of 52 cards.

$|S| = C(52, 2)$

Let E be the event of getting two cards which are diamonds.

$|E| = C(13, 2)$

$$P(E) = \frac{|E|}{|S|} = \frac{C(13,2)}{C(52,2)} = \frac{13.12}{52.51} = \frac{1}{17}.$$

41. A bag contains 6 white and 7 black balls. Altogether there are 13 balls, 2 balls are drawn at random.

Let S be the sample space.

$|S| = C(13, 2)$

Let E be the event of getting 2 white balls.

$|S| = C(6, 2)$

$$P(E) = \frac{|E|}{|S|} = \frac{C(6,2)}{C(13,2)} = \frac{6.5}{13.12} = \frac{5}{26}.$$

42. There are 100 cards which are numbered from 1 to 100.

Let S be the sample space.

$|S| = 100$.

Let E be the event of get a card whose number is divisible both by 2 and 5.

i.e. The number is divisible by 10.

E = {10, 20, 30, 40, 50, 60, 70, 80, 90, 100}

$|E| = 10$

$$P(E) = \frac{|B|}{|S|} = \frac{10}{100} = \frac{1}{10}.$$

43. 8 persons stand in a line.

Let S be the sample space.

$|S| = 8!$

Let E be the event such that two persons X and Y stand together.

Considering two persons X & Y as one there are 7 persons.

So they can stand together in 7! ways. But two persons X & Y can be arranged in 2!.

Thus $|E| = 2! \text{ x } 7!$

$$P(E) = \frac{|E|}{|S|} = \frac{2! \text{ x } 7!}{8!} = \frac{2}{8} = \frac{1}{4}.$$

Probability that two persons donot stand together is

$P(E^C) = 1 - P(E)$

$$= 1 - \frac{1}{4} = \frac{3}{4}.$$

44. A bag contains 5 white and 3 black balls. Let S be the sample sapce.

$|S| = 5 + 3 = 8$.

Let E be the event of getting a white.

$|E| = 5$

$$P)E) = \frac{|E|}{|S|} = \frac{5}{8}.$$

45. A coin is tossed 3 times.

Let S be the sample space.

$S = \{hhh, hht, hth, thh, htt, tht, tth, ttt\}$

$|S| = 8$

Let E be the event of getting atmost 2 heads.

$E = \{hht, thh, hth, htt, tht, tth, ttt\}$

$|E| = 7$

$$P(E) = \frac{|E|}{|S|} = \frac{7}{8}.$$

7. Set, Relations & Functions and Trigonometric Functions

1. $A - \bigcup_{i=1}^{n} B_i = \bigcap_{i=1}^{n}(A - B_i)$

Let $x \in A - \bigcup_{i=1}^{n} B_i$

$\Leftrightarrow x \in A - (B_1 \cup B_2 \cup .... \cup B_n)$

$\Leftrightarrow x \in A$ and $x \notin B_1 \cup B_2 \cup .... \cup B_n$

$\Leftrightarrow x \in A$ and

$(x \notin B_1$ and $x \notin B_2$ and...and $x \notin B_n)$

$\Leftrightarrow (x \in A$ and $x \notin B_1)$ and $(x \in A$ and $x \notin B_2)$

and...and $(x \in A$ and $x \notin B_n)$

$\Leftrightarrow x \in (A - B_1)$ and $x \in (A - B_2)$

and...and $x \in (A - B_n)$

$\Leftrightarrow x \in (A - B_1) \cap (A - B_2) \cap ... \cap (A - B_n)$

$\Leftrightarrow x \in \bigcap_{i=1}^{n}(A - B_i)$

$\therefore A - \bigcup_{i=1}^{n} B_i \subset \bigcup_{i=1}^{n}(A - B_i)$ ... (1)

and $\bigcap_{i=1}^{n}(A - B_i) \subset A - \bigcup_{i=1}^{n} B_i$ ... (2)

From (1) and (2), we get

$A - \bigcup_{i=1}^{n} B_i = \bigcap_{i=1}^{n}(A - B_i)$.

2. Let $x \in A - \bigcup_{i=1}^{n} B_i$

$\Leftrightarrow x \in A - (B_1 \cap B_2 \cap .... \cap B_n)$

$\Leftrightarrow x \in A$ and $x \notin (B_1 \cap B_2 \cap .... \cap B_n)$

$\Leftrightarrow x \in A$ and

$(x \notin B_1$ or $x \in B_2$ or.... or $x \notin B_n)$

$\Leftrightarrow (x \in A$ and $x \notin B_1)$ or

$(x \in A$ and $x \notin B_2)$

or...or $(x \in A$ and $x \in B_n)$

$\Leftrightarrow x \in A - B_1$ or $x \in A - B_2$

or...or $x \in A - B_n$

$\Leftrightarrow x \in (A - B_1) \cup (A - B_2) \cup ... \cup (A - B_n)$

$\Leftrightarrow x \in \bigcup_{i=1}^{n}(A - B_i)$

$\therefore A - \bigcap_{i=1}^{n} B_i \subset \bigcup_{i=1}^{n}(A - B_i)$ ... (1)

$\bigcup_{i=1}^{n}(A - B_i) \subset A - \bigcap_{i=1}^{n} B_i$ ... (2)

Form (1) and (2), we get

$A - \bigcap_{i=1}^{n} B_i = \bigcup_{i=1}^{n}(A - B_i)$.

3. Let $x \in \left(\bigcup_{i=1}^{n} A_i\right)'$

$\Leftrightarrow x \in (A_1 \cup A_2 \cup ... \cup A_n)'$

$\Leftrightarrow x \notin (A_1 \cup A_2 \cup ... \cup A_n)$

$\Leftrightarrow x \notin A_1$ and $x \notin A_2$ and ... and $x \notin A_n$

$\Leftrightarrow x \notin A_1'$ and $x \notin A_2'$ and ... $x \notin A_n'$

$\Leftrightarrow x \in A_1' \cap A_2' \cap ... \cap A_n'$

$\Leftrightarrow x \in \bigcap_{i=1}^{n} A_i'$

$\therefore \left(\bigcup_{i=1}^{n} A_i\right)' = \bigcap_{i=1}^{n} A_i'$

4.(i) Let $x \in A - (B \cup C)$

$\Leftrightarrow x \in A$ and $x \notin B \cup C$

$\Leftrightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Leftrightarrow (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$

$\Leftrightarrow x \in A - B$ and $x \in A - C$

$\Leftrightarrow x \in (A - B) \cap (A - C)$

$\therefore\ A-(B\cup C)\subset(A-B)\cap(A-C)$ ... (1)

and $(A-B)\cap(A-C)\subset A-(B\cup C)$ ... (2)

From (1) and (2), we get

$A-(B\cup C)=(A-B)\cap(A-C)$.

(ii) Let $x\in A-(B\cap C)$

$\Leftrightarrow x\in A$ and $x\notin B\cap C$

$\Leftrightarrow x\in A$ and $(x\notin B$ or $x\notin C)$

$\Leftrightarrow (x\in A$ and $x\notin B)$ or $(x\in A$ and $x\notin C)$

$\Leftrightarrow x\in A-B$ or $x\in A-C$

$\Leftrightarrow x\in(A-B)\cup(A-C)$

$\therefore\ A-(B\cap C)\subset(A-B)\cup(A-C)$ ... (1)

and $(A-B)\cup(A-C)\subset A-(B\cap C)$ ... (2)

From (1) and (2), we get

$A-(B\cap C)=(A-B)\cup(A-C)$.

5. Let E be the set of students in a school.

Let A be the set of students taking apple juice and B denote the set of students taking orange juice.

According to the questions,

$n(A)=120,\ n(B)=150$

$n(A\cap B)=100$

$\therefore n(E)=500$

Here E is the universal students.

$n(A\cup B)=n(A)+n(B)-n(A\cup B)$

= 120 + 150 - 100

= 270 - 100 = 170

Number of students taking neither apple juice nor orange juice

$=n(E)-n(A\cup B)$

= 500 - 170 = 330.

6. $R=\{(a,a^3): a$ is a prime number less than $10\}$

$=\{(2,8),(3,27),(5,125),(7,346)\}$

dom $R=\{2,3,5,7\}$

rng $R=\{8,27,125,346\}$

$R^{-1}=\{(8,2),(27,3),(125,5),(346,7)\}$

dom $R^{-1}=\{8,27,125,346\}$

rng $R^{-1}=\{2,3,5,7\}$.

7. $f(x)=\cos(\log x)$

$\therefore\ f(x^2)=\cos(\log x^2)=\cos(2\log x)$

$=\cos 2A$ where $A=\log x$

$f(y^2)=\cos(\log y^2)=\cos(2\log y)$

$=\cos 2B$ where $B=\log y$

$f(x^2y^2)=\cos\log(x^2y^2)$

$=\cos(\log x^2+\cos y^2)$

$=\cos(2\log x+2\log y)$

$=\cos(2A+2B)$.

$f\left(\frac{x^2}{y^2}\right)=\cos\left(\log\frac{x^2}{y^2}\right)$

$=\cos\left(\log x^2-\log y^2\right)$

$=\cos\left(2\log x-2\log y\right)$

$=\cos\left(2A-2B\right)$.

$\therefore\ \frac{1}{2}\left[f\left(\frac{x^2}{y^2}\right)+f(x^2y^2)\right]$

$=\frac{1}{2}.[\cos(2A-2B)+\cos(2A+2B)]$

$=\frac{1}{2}\ .\ 2\cos 2A\ \cos 2B$

$=\cos 2A\ \cos 2B$.

L.H.S. $=f(x^2).f(y^2)-\frac{1}{2}\left\{f\left(\frac{x^2}{y^2}\right)+f(x^2y^2)\right\}$

= cos 2A . cos 2B - cos 2A . cos 2B

= 0.

8. Given that $f(x) = \log_e\left(\frac{1+x}{1-x}\right)$

$$\therefore\ f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\frac{(1+x)^2}{(1-x)^2}$$

$$= \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right) = 2f(x).$$

9. Given that

$$f(n) = \begin{cases} 0 & \text{when } n = 1 \\ f\left(\left[\frac{n}{2}\right]\right)+1 & \text{when } n > 1 \end{cases}$$

$$\therefore f(35) = f\left(\left[\frac{35}{2}\right]\right)+1$$

$$= f([17.5])+1$$

$$= f(17)+1$$

$$= f\left(\left[\frac{17}{2}\right]\right)+1+1$$

$$= f([8.5])+2$$

$$= f(8)+2$$

$$= f\left(\left[\frac{8}{2}\right]\right)+1+2$$

$$= f(4)+3$$

$$= f\left(\left[\frac{4}{2}\right]\right)+1+3$$

$$= f([2])+4$$

$$= f(2)+4$$

$$= f\left(\left[\frac{2}{2}\right]\right)+1+4$$

$$= f(1)+5 = 0+5 = 5.$$

10. Given that

$$a\,f(x) + b\,f\left(\frac{1}{x}\right) = \frac{1}{x} - 5 \quad \text{... (1)}$$

writing $\frac{1}{x}$ for x, we get

$$a\ f\left(\frac{1}{x}\right) + bf(x) = x - 5 \quad \text{... (2)}$$

Adding (1) and (2), we get

$$a\left[f(x)+f\left(\frac{1}{x}\right)\right] + b\left[f\left(\frac{1}{x}\right)+f(x)\right] = x + \frac{1}{x} - 10$$

$$\Rightarrow (a+b)\left[f(x)+f\left(\frac{1}{x}\right)\right] = x + \frac{1}{x} - 10.$$

$$\Rightarrow f(x)+f\left(\frac{1}{x}\right) = \frac{1}{a+b}\left[x+\frac{1}{x}-10\right] \quad \text{... (3)}$$

Subtracting (2) from (1), we get

$$a\left[f(x)-f\left(\frac{1}{x}\right)\right] - b\left[f(x)-f\left(\frac{1}{x}\right)\right] = \frac{1}{x} - x$$

$$\Rightarrow (a-b)\left[f(x)-f\left(\frac{1}{x}\right)\right] = \frac{1}{x} - x$$

$$\Rightarrow f(x)-f\left(\frac{1}{x}\right) = \frac{1}{a-b}\left(\frac{1}{x}-x\right) \quad \text{... (4)}$$

Adding (3) and (4), we get

$$2f(x) = \frac{1}{a+b}\left[x+\frac{1}{x}-10\right] + \frac{1}{a-b}\left(\frac{1}{x}-x\right)$$

$$\Rightarrow f(x) = \frac{1}{2(a+b)}\left[x+\frac{1}{x}-10\right] + \frac{1}{2(a-b)}\left(\frac{1}{x}-x\right).$$

11. $\text{L.H.S.} = \left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$$= \left(\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}\right)^n$$

$$+ \left(\frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}}\right)^n$$

$$= \left(\cot\frac{A-B}{2}\right)^n + \left(-\cot\frac{A-B}{2}\right)^n$$

$$= \cot^n\frac{A-B}{2} + (-1)^n .\ \cot^n\frac{A-B}{2} \qquad \ldots (1)$$

When n is even, $(-1)^n = 1$ then (1) becomes

$$\cot^n\frac{A-B}{2} + \cot^n\frac{A-B}{2} = 2\cot^n\frac{A-B}{2}$$

When n is odd, $(-1)^n = -1$.
Then (1) becomes

$$\cot^n\frac{A-B}{2} - \cot^n\frac{A-B}{2} = 0.$$

12. Given that $\tan\beta = \dfrac{n^2\sin\alpha\cos\alpha}{1-n^2\sin^2\alpha}$

L.H.S. $= \tan(\alpha - \beta)$

$$= \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha} - \frac{n^2\sin\alpha\cos\alpha}{1-n^2\sin^2\alpha}}{1+\frac{\sin\alpha}{\cos\alpha}.\frac{n^2\sin\alpha\cos\alpha}{1-n^2\sin^2\alpha}}$$

$$= \frac{\sin\alpha(1-n^2\sin^2\alpha) - n^2\sin\alpha\cos^2\alpha}{\cos\alpha(1-n^2\sin^2\alpha) + n^2\sin^2\alpha\cos\alpha}$$

$$= \frac{\sin\alpha - n^2\sin^3\alpha - n^2\sin\alpha\ \cos^2\alpha}{\cos\alpha - n^2\sin^2\alpha\cos\alpha + n^2\sin^2\alpha\cos\alpha}$$

$$= \frac{\sin\alpha - n^2\sin\alpha(\sin^2\alpha + \cos^2\alpha)}{\cos\alpha}$$

$$= \frac{(1-n^2)\sin\alpha}{\cos\alpha} = (1-n^2)\tan\alpha.$$

13. The given equation is $3\sin^2 x - 7\sin x + 2 = 0$

$\Rightarrow 3\sin^2 x - 6\sin x - \sin x + 2 = 0$

$\Rightarrow 3\sin x - (\sin x - 2) - 1(\sin x - 2) = 0$

$\Rightarrow (\sin x - 2)(3\sin x - 1) = 0$

Here $\sin x - 2 = 0 \Rightarrow \sin x = 2$ which is rejected.
When $3\sin x - 1 = 0$

$\Rightarrow \sin x = \dfrac{1}{3} = \sin\alpha$ (say)

$\Rightarrow x = n\pi + (-1)^n\alpha$ where n = 0, 1, 2, 3, 4, 5.
The number of values of x is 6.

14. Given that

$$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

$$\Rightarrow \frac{1}{a+c} + \frac{1}{b+c} = \frac{1}{a+b+c} + \frac{1}{a+b+c} + \frac{1}{a+b+c}$$

$$\Rightarrow \frac{1}{a+c} - \frac{1}{a+b+c} + \frac{1}{b+c} - \frac{1}{a+b+c} = \frac{1}{a+b+c}$$

$$\Rightarrow \frac{a+b+c-a-c}{(a+c)(a+b+c)} + \frac{a+b+c-b-c}{(b+c)(a+b+c)} = \frac{1}{a+b+c}$$

$$\Rightarrow \frac{b}{a+c} + \frac{a}{b+c} = 1$$

$$\Rightarrow \frac{b(b+c)+a(a+c)}{(a+c)(b+c)} = 1$$

$$\Rightarrow \frac{b^2+bc+a^2+ac}{ab+ac+bc+c^2} = 1$$

$\Rightarrow a^2 + b^2 + bc + ac = ab + ac + bc + c^2$

$\Rightarrow a^2 + b^2 - c^2 = ab$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = \frac{1}{2}$$

$\Rightarrow \cos C = \cos 60^0$

$\Rightarrow C = 60.$

15. Let $a = x^2 + x + 1$, $b = 2x + 1$, $c = x^2 - 1$.

All three sides are +ve.

$\therefore\ b > 0$ and $c > 0$

$\Rightarrow 2x + 1 > 0$ and $x^2 - 1 > 0$

$\Rightarrow x > -\frac{1}{2}$ and $x > 1$ or $x < -1$

$\Rightarrow x > 1$

$\Rightarrow x^2 > x$

$\Rightarrow x^2 + x > 2x$

$\Rightarrow x^2 + x + 1 > 2x + 1 > 0$

$\Rightarrow$ Here a > b also a > c

So A is greatest angle

$$\text{Cos A} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(2x+1)^2 + (x^2-1)^2 - (x^2+x+1)^2}{2(2x+1)(x^2-1)}$$

$$= \frac{4x^2 + 4x + 1 + x^4 - 2x^2 + 1 - x^4 - x^2 - 1 - 2x^3 - 2x^2 - 2x}{2(2x^3 - 2x + x^2 - 1)}$$

$$= \frac{-2x^3 - x^2 + 2x + 1}{2(2x^3 + x^2 - 2x - 1)}$$

$$= \frac{-(2x^3 - x^2 - 2x - 1)}{2(2x^3 + x^2 - 2x - 1)} = -\frac{1}{2}$$

$\Rightarrow A = 120^0$.

8. Principle of Mathematical Induction, Complex numbers, Linear Inequalities, Permutations and Combinations, Binomial Theorem, Sequence and Series.

1. Let P (n) be the statement :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

First we shall show that P(1) is true.

$$\text{L.H.S. of P(1)} = \frac{1}{1.2} = \frac{1}{2}$$

$$\text{R.H.S. of P(1)} = \frac{1}{1+1} = \frac{1}{2}$$

L.H.S. of P(1) = R.H.S. of P(1)

So P(1) is true.

Let P(k) be true.

$$\text{i.e. } \frac{1}{1\,.\,2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{k(k+1)} = \frac{k}{k+1}.$$

We shall have to show that P(k + 1) is true.

L.H.S. of P (k + 1)

$$= \frac{1}{1\,.\,2} + \frac{1}{2.3} + ... + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2} = \text{R.H.S. of P(k + 1).}$$

So P (k + 1) is true.

Here we see that

(i) P (1) is true

(ii) P (k) is true $\Rightarrow p(k+1)$ is true.

So according to the principle of induction, P(n) is true for all $n \in N$.

$$\text{i.e. } \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

2. Let P(n) be the statement

"$n^3 - n$ is divisible by 3".

First we shall show that P(1) is true.

Taking n = 1, we get $1^3 - 1 = 1 - 1 = 0$

which is divisible by 3

So P(1) is true.

Let P(k) be true.

$\Rightarrow k^3 - k$ is divisible by 3

$\Rightarrow k^3 - k = 3m$ where $n \in Z$

We shall have to show that P (k + 1) is true.

$\therefore (k+1)^3 - (k+1)$

$= k^3 + 3k^2 + 3k + 1 - k - 1$

$= (k^3 - k) + 3k^2 + 3k$

$= 3m + 3k^2 + 3k$

$= 3(m + k^2 + k)$ which is divisible by 3

So P(k+1) is true.

Here we see that

(i) P(1) is true

(ii) P(k) is true $\Rightarrow P(k+1)$ is true

So according to the principle of induction, P(n) is true for all $n \in N$.

Thus $n^3 - n$ is divisible by 3 for all $n \in N$.

3. We know $1 + w + w^2 = 0$ and $w^3 = 1$

L.H.S.

$= (1-w+w^2)(1-w^2+w^4)(1-w^4+w^8)(1-w^8+w^{16})$ ... to 2n factors.

$=(1-w+w^2)(1-w^2+w^3.w)(1-w^3.w+w^6.w^2)(1-w^6.w^2+w^{15}.w)$ ... to 2n factors.

$= (1-w+w^2)(1-w^2+w)(1-w+w^2)(1-w^2+w)$ ... to 2n factors.

$= (1+w^2-w)(1+w-w^2)(1+w^2-w)(1+w-w^2)$... to 2n factors.

$= (-w-w)(-w^2-w^2)(-w-w)(-w^2-w^2)$ ... to 2n factors.

$= (-2w)(-2w^2)(-2w)(-2w^2)$ ... to 2n factors

$= (-2w)(-2w)$ ... to n factors

$(-2w^2)(-2w^2)$... to n factors

$= (-2w)^n (-2w^2)^n$

$= (-1)^n . 2^n . w^n . (-1)^n . 2^n . w^{2n}$

$= (-1)^{2n} . 2^{2n} . w^{3n} = 1 . 2^{2n} . 1 = 2^{2n}$.

4. Given that $z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1 = 0$

$\Rightarrow (z_1 + z_2w + z_3w^2)(z_1 + z_2w^2 + z_3w) = 0$

$\Rightarrow z_1 + z_2w + z_3w^2 = 0$ or

$z_1 + z_2w^2 + z_3w = 0$

$\Rightarrow z_1 + z_2w - z_3(1+w) = 0$ or

$z_1 - z_2(1+w) + z_3w = 0$

$\Rightarrow (z_1 - z_3) + (z_2 - z_3)w = 0$ or

$(z_1 - z_2) - (z_2 - z_3)w = 0$

$\Rightarrow z_3 - z_1 = (z_2 - z_3)w$ or

$z_1 - z_2 = (z_2 - z_3)w$

$\Rightarrow |z_3 - z_1| = |w||z_2 - z_3|$ or

$|z_1 - z_2| = |w||z_2 - z_3||w|$

$\Rightarrow |z_3 - z_1| = |z_2 - z_3|$ or $|z_1 - z_2| = |z_2 - z_3|$

$\Rightarrow |z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$.

5. Given that $z = x + iy$

$\Rightarrow |z| = |x + iy| = \sqrt{x^2 + y^2}$

We know $(|x| - |y|)^2 \geq 0$

$\Rightarrow |x|^2 + |y|^2 - 2|x||y| \geq 0$

$\Rightarrow |x|^2 + |y|^2 \geq 2|x||y|$

$\Rightarrow x^2 + y^2 \geq 2|x||y|$

$\Rightarrow 2(x^2 + y^2) \geq x^2 + y^2 + 2|x||y|$

$\Rightarrow 2|z|^2 \geq |x|^2 + |y|^2 + 2|x||y|$

$\Rightarrow 2|z|^2 \geq (|x| + |y|)^2$

$\Rightarrow (|x| + |y|)^2 \leq 2|z|^2$

$\Rightarrow |x| + |y| \leq \sqrt{2}|z|$.

6. The given digits are 1, 2, 3, 4, 5, 6, 8.

We shall form integers with distinct digits that exceed 5500.

The numbers may be of 7 digits, 6 digits, 5 digits and 4 digits but greater than 5500.

Total number of integers of 7 digits

= P(7, 7) = 7! = 5040.

Total number of integers of 6 digits

$= P(7, 6) = P(7,6) = \frac{7!}{1!} = 5040$.

Total number of integers of 5 digits

$= P(7,5) = \frac{7!}{(7-5)!} = \frac{7!}{2!} = 7.6.5.4.3 = 2520$.

4 digit numbers : In this case 1000th place is filled up by 5 or 6 or 8.

When 1000th place is filled up by 5, then 100th place is filled up by 6 or 8 in 2 ways 10th place is filled up in 5 ways and unit place is filled up in 4 ways.

The number of 4 digit numbers shorting with 5 and greater than 5500 = 1 x 2 x 5 x 4 = 40.

When 1000th place is filled up by 5, then it can be filled up in 2 ways (i.e. by 6 or 8).

100th place is filled up in 6 ways and 10th place in 5 ways and unit place in 4 ways.

$\therefore$ 4 digit integers greater than 5500 and not starting with 5 = 2 x 6 x 5 x 4 = 240

Total 4-digit integers greater than

5500 = 40 + 240 = 280.

Total integers greater than

5500 = 5040 + 5040 + 2520+280 = 12880.

7. The given word is "FAILURE". In the arrangements, the voweles are kept together.

Considering the voweles "AIUE" as and thing, we have 4 things (AIUE), F, L, R.

The number of arrangement of these 4 things

= P(4, 4) = 4!.

Again 4-voweles can be arranged among themselves in P(4, 4) or 4! ways.

So total number of arrangements

= 4! x 4! = 24 x 24 = 576.

8. Given that $n_{p_r} = 1680$ and $n_{c_r} = 70$

We know $n_{c_r} = \frac{n_{p_r}}{r!}$

$\Rightarrow 70 = \frac{1680}{r!}$

$\Rightarrow r! = \frac{1680}{70} = 24 = 4!$

$\Rightarrow r = 4.$

$\Rightarrow n_{p_r} = 1680$

$\Rightarrow n(n-1)(n-2)(n-3) = 8.7.6.5$

$\Rightarrow n = 8$

$\therefore n = 8, r = 4$.

9. These are 6 gentelemen and 4 ladies. A committee of 5 is to be selected so as to include at least one lady. The following types of selection can be made.

(i) 1 lady and 4 gentlemen.

(ii) 2 ladies and 3 gentlemen.

(iii) 3 ladies and 2 gentlemen

(iv) 4 ladies and 1 gentleman.

Case - I : 1 lady is selected from 4 ladies in C(4,1) way. After each time the selection of made, 4 gentlemen can be selected out of 6 gentlemen in C(6, 4) ways.

Total number of these types of committee

$= C(4,1) \times C(6,4)$

$= 4.\frac{6.5.4.3}{4.3.2.1} = 4 \times 15 = 60$.

Case - II : 2 ladies and 3 gentlemen are selected in C(4, 2) x C (6, 3) ways

$= \frac{4.3}{2.1} \times \frac{6.5.4}{3.2.1}$

$= 6 \times 20 = 120$.

Case - III : 3 ladies and 2 gents are selected in C (4, 3) x C (6, 2)

$= \frac{4.3.2}{3.2.1} \times \frac{6.5}{2.1}$

$= 4 \times 15 = 60$

Case - IV : 4 ladies and 1 gentleman are selected in C (4, 4) x C (6, 1).

Total number of selection

= 60 + 120 + 60 + 6 = 246.

10. A bag contains 4 black balls and 5 white balls. 6 balls are drawn arbitraily. Altogether there are 4 + 5 = 9 balls. The number of ways 6 balls are drawn

$= C(9,6) = \frac{9!}{6!\,3!} = \frac{9.\,8.\,7}{3.2.1} = 84$.

11. We know

$C_0 + C_1x + C_2x^2 + ... + C_rx^r + C_{r+1}x^{r+1}$

$+C_{r+2}x^{r+2} + ... + C_nx^n = (1+x)^n$ ... (1)

Also we get

$C_0x^n + C_1x^{n-1} + C_2x^{n-2} + ...$

$+C_rx^{n-r} + C_{r+1}x^{n-r-1} + C_{r+2}x^{n-r-2} + ...$

$+C_n = (x+1)^n$ ... (2)

On multiplication, we get

$(C_0 + C_1x + C_2x^2 + ... + c_rx^r + C_{r+1}x^{r+1}$

$+C_{r+2}x^{r+2} + ... + C_nx^n)$

$\times (C_0x^n + C_1x^{n-1} + C_2x^{n-2} + ...$

$+C_rx^{n-r} + C_{r+1}x^{n-r-1} + C_{r+2}x^{n-r-2} + ... + C_n)$

$= (1+x)^n (x+1)^n = (1+x)^{2n}$

$= {}^{2n}C_0 + {}^{2n}C_1x + {}^{2n}C_2x^2 + ...$

$+ {}^{2n}C_{n-r}x^{n-r} + ... + {}^{2n}C_{2n}x^{2n}$ ... (3)

Coefficient of x^{n-r} from the left side of (3)

$= C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n$

Coefficient of x^{n-r} from the right side of (3)

$= {}^{2n}C_{n-r}$.

Equaling the coefficient of x^{n-r} from both sides of (3), we get.

$C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ...$

$+C_{n-r}C_n = {}^{2n}C_{n-r}$

$= \frac{(2n)!}{(n-r)!\{2n-(n-r)\}!}$

$= \frac{(2n)!}{(n-r)!(n+r)!}$.

12. Let a be the 1[st] term and r be the common ratio of G.P.

$S = a + ar + ar^2 + ... + ar^{n-1}$

$= a\frac{(1-r^n)}{1-r}$

$P = a.ar.ar^2 ... ar^{n-1}$

$= a^n . r^{1+2+3+...+(n-1)}$

$= a^n\, r^{\frac{n(n-1)}{2}}$

$P^2 = a^{2n}\, r^{n(n-1)}$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + ... + \frac{1}{ar^{n-1}}$

$= \frac{1}{a}\left(\frac{1-\frac{1}{r^n}}{1-\frac{1}{r}}\right) = \frac{r^n - 1}{ar^n} \times \frac{r}{r-1} = \frac{r^n - 1}{ar^{n-1}(r-1)}$.

$$\frac{S}{R}=\frac{\frac{a(1-r^n)}{1-r}}{\frac{r^n-1}{ar^{n-1}(r-1)}}=\frac{a(1-r^n)}{1-r}\text{ x }\frac{ar^{n-1}(r-1)}{r^{n-1}}$$

$$=a^2r^{n-1}$$

$$\therefore\left(\frac{S}{R}\right)^n=a^{2n}.r^{n(n-1)}=p^2.$$

13. The given series is

$0.7 + 0.77 + 0.777 + ... +$ to n terms

$= 7 \text{ x } 0.1 + 7 \text{ x } 0.11 + 7 \text{ x } 0.111 + ...$ to n terms

$= 7(0.1 + 0.11 + 0.111 + ...$ to n terms)

$$=\frac{7}{9}\,(0.9+0.99+0.999+\ldots\text{ to n terms})$$

$$=\frac{7}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots\text{to n terms}\right]$$

$$=\frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+\ldots\text{to n terms}\right]$$

$$=\frac{7}{9}\left[n-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\ldots\text{to n terms}\right)\right]$$

$$=\frac{7}{9}\left[n-\frac{1}{10}\frac{\left\{1-\left(\frac{1}{10}\right)^n\right\}}{1-\frac{1}{10}}\right]$$

$$=\frac{7}{9}\left[n-\frac{1}{9}\left\{1-\left(\frac{1}{10}\right)^n\right\}\right]$$

$$=\frac{7}{81}\left[9n-1+\frac{1}{10^n}\right].$$

14. Let t_n be the n^{th} term.

$$t_n=\frac{n^4}{n!}=\frac{n^3}{(n-1)!}=\frac{(n^3-1)+1}{(n-1)!}$$

$$=\frac{(n-1)(n^2+n+1)+1}{(n-1)!}$$

$$=\frac{(n-1)(n^2+n+1)}{(n-1)!}+\frac{1}{(n-1)!}$$

$$=\frac{n^2+n+1}{(n-2)!}+\frac{1}{(n-1)!}$$

$$=\frac{(n-2)(n+3)+7}{(n-2)!}+\frac{1}{(n-1)!}$$

$$=\frac{n+3}{(n-3)!}+\frac{7}{(n-2)!}+\frac{1}{(n-1)!}$$

$$=\frac{(n-3)+6}{(n-3)!}+\frac{7}{(n-2)!}+\frac{1}{(n-1)!}$$

$$=\frac{1}{(n-4)!}+\frac{6}{(n-3)!}+\frac{7}{(n-2)!}+\frac{1}{(n-1)!}$$

The given series $=\sum t_n$

$$=\sum\frac{1}{(n-4)!}+6\sum\frac{1}{(n-3)!}+7\sum\frac{1}{(n-2)}+\sum\frac{1}{(n-1)!}$$

$$=e+6e+7e+e$$

$$=15e$$

15. Given equation is $ax^2 + bx + c = 0$

α and β are two roots

$$\alpha+\beta=-\frac{b}{a},\ \alpha\beta=\frac{c}{a}$$

$$\log_e(a-bx+cx^2)=\log_e a\left[1-\frac{b}{a}x+\frac{c}{a}x^2\right]$$

$$=\log_e a[1+(\alpha+\beta)x+\alpha\beta.x^2]$$

$$=\log_e a(1+\alpha x)(1+\beta x)$$

$$=\log_e a+\log_e(1+\alpha x)+\log_e(1+\beta x)$$

$$=\log_e a+\alpha x-\frac{d^2x^2}{2}+\frac{\alpha^3x^3}{3}-\ldots$$

$$+\beta x-\frac{\beta^2x^2}{2}+\frac{\beta^3x^3}{3}-\ldots$$

$$=\log_e a+(\alpha+\beta)x-\frac{1}{2}(\alpha^2+\beta^2)x^2$$

$$+\frac{1}{3}(\alpha^3+\beta^3)x^3$$

9. Straight line, Conic Section, Introduction to three-dimensional Geometry.

1. The given lines are

$2x + y - 3 = 0$ ... (1)

$5x + ky - 3 = 0$ ... (2)

$3x - y - 2 = 0$ ... (3)

Solving (1) and (3), we get

$$\frac{x}{-2-3} = \frac{y}{-9+4} = \frac{1}{-2-3}$$

$$\Rightarrow \frac{x}{-5} = \frac{y}{-5} = \frac{1}{-5}$$

$$\Rightarrow x = 1,\ y = 1.$$

The coordinates of the point of intersecton of (1) and (3) is (1, 1).

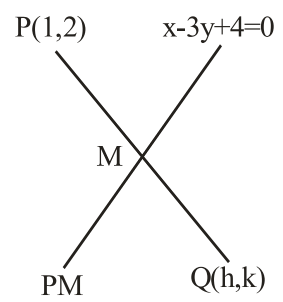
Since the given lines are concurrent, the point (1,1) will satisfy the equation (2)

$$\therefore 5.1 + k.1 - 3 = 0$$

$$\Rightarrow k + 2 = 0$$

$$\Rightarrow k = -2.$$

2. The given line is $x - 3y + 4 = 0$ ... (1)



Let P be the given point whose coordinates are (1, 2).

From let us draw a perpendicular to the line. Let us produce PM to Q such that PM = MQ.

Here Q is the image of P. Let the coordinates Q be (h, k). Since M is the middle point of PQ, its coordinates are $\left(\frac{h+1}{2}, \frac{k+2}{2}\right)$.

The slope of the line (1) $= -\frac{1}{(-3)} = \frac{1}{3}$

Slope of the line $PQ = \frac{k-2}{h-1}$.

Since PQ is perpendicular to the line (1),

We have $\frac{k-2}{h-1} \times \frac{1}{3} = -1$

$$\Rightarrow k - 2 = -3(h-1)$$

$$\Rightarrow k - 2 = -3h + 3$$

$$\Rightarrow 3h + k = 5 \quad \text{... (2)}$$

Again the point $\left(\frac{h+1}{2}, \frac{k+2}{2}\right)$ lies on the line (1).

$$\therefore \frac{h+1}{2} - 3\left(\frac{k+2}{2}\right) + 4 = 0$$

$$\Rightarrow h + 1 - 3(k+2) + 8 = 0$$

$$\Rightarrow h - 3k + 3 = 0$$

$$\Rightarrow h - 3k = -3 \quad \text{... (3)}$$

Solving (2) and (3), we get

$$h = \frac{6}{5},\ k = \frac{7}{5}$$

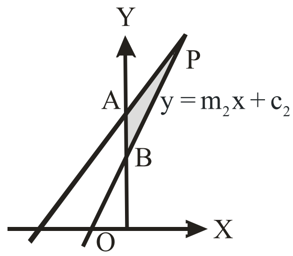
The coordinates of the image Q are $\left(\frac{6}{5}, \frac{7}{5}\right)$.

3. Three lines are

$y = m_1x + c_1$ ... (1)

$y = m_2x + c_2$ ... (2)

and $x = 0$ ... (3)



Let the line (1) and (2) intersect y-axis at A and B respectively.

$OA = c_1,\ OB = c_2$

The coordinates of A and B are (O, c_1) and (O, c_2) respectively.

Solving (1) and (2), we get

$m_2x + c_2 = m_1x + c_1$

$\Rightarrow (m_2 - m_1)x = c_1 - c_2$

$\Rightarrow x = \dfrac{c_1 - c_2}{m_2 - m_1}$

From (1), we get

$y = m_1 . \dfrac{c_1 - c_2}{m_2 - m_1} + c_1$

$= \dfrac{m_1(c_1 - c_2) + c_1(m_2 - m_1)}{m_2 - m_1}$

$= \dfrac{m_1c_1 - m_1c_2 + m_2c_1 - m_1 - c_1}{m_2 - m_1}$

$= \dfrac{m_2c_1 - m_1c_2}{m_2 - m_1}$.

The coordinates of P are

$\left(\dfrac{c_1 - c_2}{m_2 - m_1}, \dfrac{m_2c_1 - m_1c_2}{m_2 - m_1}\right)$

Area of the triangle PAB

$= \dfrac{1}{2}\left[\dfrac{c_1 - c_2}{m_2 - m_1}(c_1 - c_2) + 0\left(c_2 \dfrac{m_2c_1 - m_1c_2}{m_2 - m_1}\right)\right.$

$\left. + 0\left(\dfrac{c_1 - c_2}{m_2 - m_1} c_1\right)\right]$

$= \dfrac{1}{2} \dfrac{(c_1 - c_2)^2}{m_2 - m_1}$

$= \dfrac{1}{2} \dfrac{(c_1 - c_2)^2}{|m_2 - m_1|}$.

4. The equation of the line is $3x - 4y - 5 = 0$ ... (1)

The equation of a line paralled to the above line is 3x - 4y + k = 0 ... (2)

Putting x = -1 in (1), we get

$-3 - 4y - 5 = 0$

$\Rightarrow 4y = -8$

$\Rightarrow y = -2$

A point on the line (1) is (-1, -2).

The length of the perpendicular form (-1, -2) to the line (2) = 1

$\Rightarrow \dfrac{|3(-1) - 4(-2) + k|}{\sqrt{3^2 + (-4)^2}} = 1$

$\Rightarrow \dfrac{|5 + k|}{5} = 1$

$\Rightarrow 5 + k = \pm 5$

$\Rightarrow x = 0, -10$.

The required lines are

3x - 4y = 0 and 3x - 4y - 10 = 0

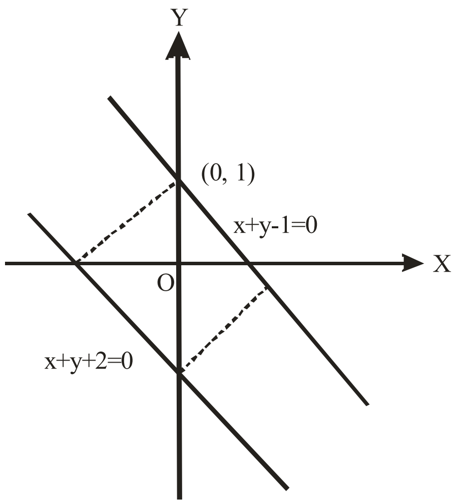
5. Two given lines are

x + y - 1 = 0 ... (1)

x + y + 2 = 0 ... (2)

Putting x = 0 in (1), we get y = 1.

The point (0, 1) lines on the line (1).



The distance between the parallel lines

= The length of the perpendicular form (0, 1) to the line (2)

$= \dfrac{|0 + 1 + 2|}{\sqrt{1^2 + 1^2}} = \dfrac{3}{\sqrt{2}}$.

$\Rightarrow$ The length of the side of the square $= \dfrac{3}{\sqrt{2}}$.

Area of the square $= \left(\dfrac{3}{\sqrt{2}}\right)^2 = \dfrac{9}{2}$ sq. unit.

6. Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2 + y + c = 0 \quad \text{... (1)}$$

Its centre is (-g, -f).

The given line is $4x + y = 16$ ... (2)

Since the centre (-g, -f) lies on the line (2), we have

$$4(-g) + (-f) = 16$$

$$\Rightarrow 4g + f = -16 \quad \text{... (3)}$$

Since the cirlce (1) is passing through (4, 1) and (6, 5), we get

$$4^2 + 1^2 + 2g.4 + 2f.1 + c = 0$$

$$\Rightarrow 16 + 1 + 8g + 2f + c = 0$$

$$\Rightarrow 8g + 2f + c + 17 = 0 \quad \text{... (4)}$$

and $6^2 + 5^2 + 2g.6 + 2f.f + c = 0$

$$\Rightarrow 12g + 10f + c + 61 = 0 \quad \text{... (5)}$$

Subtracting (4) from (5), we get

$$4g + 8f + 44 = 0$$

$$\Rightarrow 4g + 8f = -44 \quad \text{... (6)}$$

Subtracting (3) from (6), we get

$$7f = -28$$

$$\Rightarrow f = -4$$

From (3), we get

4g - 4 = -16

$\Rightarrow$ 4g = -12

$\Rightarrow$ g = -3

Again from (4), we get

8(-3) + 2(-4) + c + 17 = 0

$\Rightarrow$ - 24 - 8 + c + 17 = 0

$\Rightarrow$ c = 15

$\therefore$ g = -3, f = -4, c = 15.

Required circle is

$$x^2 + y^2 + 2g(-3)x + 2(-4)y + 15 = 0$$

$$\Rightarrow x^2 + y^2 - 6x - 8y + 15 = 0.$$

7. Centre is on x - axis.

Let the centre be (h, 0).

The distance from the centre (h, 0) to the point (2, 3) = 5.

$$\Rightarrow \sqrt{(h-2)^2 + (0-3)^2} = 5$$

$$\Rightarrow (h-2)^2 + 9 = 25$$

$$\Rightarrow 4^2 - 4h + 4 + 9 - 25 = 0$$

$$\Rightarrow 4^2 - 4h - 12 = 0$$

$$\Rightarrow (h-6)(h+2) = 0$$

$$\Rightarrow h = 6, -2.$$

Centre is (6, 0) and (-2, 0).

One circle is $(x-6)^2 + y^2 = 5^2$

$$\Rightarrow x^2 - 12x + 36 + y^2 = 25$$

$$\Rightarrow x^2 + y^2 - 12x + 11 = 0$$

Another circle is

$$(x+2)^2 + (y-0)^2 = 5^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 = 25$$

$$\Rightarrow x^2 + y^2 + 4x - 21 = 0.$$

8. The equation of the parabola whose axis is parallel to x-axis is

$$y^2 + ax + by + c = 0 \quad \text{... (1)}$$

The parabola is passing throug (1, 2), (-2, 3) and (2, -1).

Since the parabola (1) is passing through (1, 2), we have

4 + a . 1 + b . 2 + c = 0

$\Rightarrow$ a + 2b + c + 4 = 0 ... (2)

Since it a passing through (-2, 3), we have

$9 + a(-2) + b \cdot 3 + c = 0$

$\Rightarrow -2a + 3b + c + 9 = 0$ ... (3)

Again $(-1)^2 + a \cdot 2 + b(-1) + c = 0$

$\Rightarrow 2a - b + c + 1 = 0$ ... (4)

Subtracting (2) and (3), we have

$-2a - a + 3b - 2b + 9 - 4 = 0$

$\Rightarrow -3a + b + 5 = 0$ ... (5)

Subtracting (3) from (4), we get

$4a - 4b - 8 = 0$

$\Rightarrow a - b - 2 = 0$ ... (6)

Adding (5) and (6), we get

$-2a + 3 = 0$

$\Rightarrow a = \frac{3}{2}$.

From (6), we get

$\frac{3}{2} - b - 2 = 0$

$\Rightarrow b = \frac{3}{2} - 2 = -\frac{1}{2}$

From (2), we get $\frac{3}{2} + 2\left(-\frac{1}{2}\right) + c + 4 = 0$

$\Rightarrow \frac{3}{2} - 1 + c + 4 = 0$

$\Rightarrow c = 1 - \frac{3}{2} - 4 = \frac{2-3-8}{2} = -\frac{9}{2}$

Equation of the parabola is

$x^2 + \frac{3}{2}x - \frac{1}{2}y - \frac{9}{2} = 0$

$\Rightarrow 2x^2 + 3x - y - 9 = 0$

9. The equation of the ellipse is

$16x^2 + 25y^2 - 32x + 50y - 359 = 0$ ...(1)

$\Rightarrow 16(x^2 - 2x + 1) + 25(y^2 + 2y + 1)$

$= 359 + 16 + 25$

$\Rightarrow 16(x-1)^2 + 25(y+1)^2 = 400$

$\Rightarrow \frac{(x-1)^2}{25} + \frac{(y+1)^2}{16} = 1$

$\Rightarrow \frac{(x-1)^2}{5^2} + \frac{[y-(-1)]^2}{4^2} = 1$

$a = 5, b = 4$

$c^2 = a^2 - b^2 = 25 - 16 = 9$

$\Rightarrow c = 3$

eccentricity $e = \frac{c}{a} = \frac{3}{5}$.

Centre of the ellipse a $(h, k) = (1, -1)$.

The coordinates of the foci are

$(\pm c + h, 0 + k) = (\pm 3 + 1, 0 - 1)$

$= (4, -1)$ and $(-2, -1)$.

Coordinates of the vertices are

$(\pm a + h, 0 + k) = (\pm 5 + 1, 0 - 1)$

$= (6, -1) \& (-4, -1)$

Length of the later rectum

$= \frac{2b^2}{a} = 2 \cdot \frac{16}{5} = \frac{32}{5}$

End points of the later recta are

$(\pm c + h, \pm \frac{b^2}{a} + k) = (\pm 3 + 1, \pm \frac{16}{5} - 1)$

$= \left(4, \frac{11}{5}\right), \left(4, -\frac{21}{5}\right), \left(-2, \frac{11}{5}\right), \left(-2, -\frac{21}{5}\right)$.

Equation of the major axis is

$y = k \Rightarrow y = -1$.

Equation of the minor axis is

$x = h \Rightarrow x = 1$.

Equation of the directrices are

$$x = \pm\frac{a^2}{c} + h$$

$$\Rightarrow x = \pm\frac{25}{3} + 1$$

$\Rightarrow 3x - 28 = 0$ and $3x + 22 = 0$.

10. Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{... (1)}$$

Given that the coordinates of the foci are $(\pm, 0)$.

i.e. $(\pm c, 0) = (\pm 6, 0) \Rightarrow c = 6$.

Length of the latus rectum

$$= \frac{2b^2}{a} = 10$$

$\Rightarrow b^2 = 5a$

Also $c^2 = a^2 + b^2$

$\Rightarrow 36 = a^2 + 5a$

$\Rightarrow a^2 + 5a - 36 = 0$

$\Rightarrow (a+9)(a-4) = 0$

$\Rightarrow a = 4 \quad (\because a > 0,\ a = -9$ is rejected)

$b^2 = 5.4 = 20$

The required equation of the hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

$\Rightarrow 5x^2 - 4y^2 = 80$.

11. Here ABC is a triangle.

The coordinates of A, B and C are (2a, 2, 6), (-4, 3b, 10) and (8, 14, 2c).

The coordiantes of the centroid of the triangle ABC are

$$\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6+10+2c}{3}\right)$$

$$= \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c+16}{3}\right)$$

Given that the centroid of the triangle ABC is origin.

$$\therefore \frac{2a+4}{3} = 0, \frac{3b+16}{3} = 0, \frac{2c+16}{3} = 0$$

$\Rightarrow 2a + 4 = 0,\ 3b + 16 = 0,\ 2c + 16 = 0$

$\Rightarrow a = 2,\ b = -\frac{16}{4},\ c = -8$.

12. Let P be a point on y-axis. Let the coordinates of P be (o, y, 0).

Let Q be the point (3, -2, 5).

Given that $OP = 5\sqrt{2}$

$\Rightarrow \sqrt{(3-0)^2 + (-2-y)^2 + (5-0)^2} = 5\sqrt{2}$

$\Rightarrow 9 + (-2-y)^2 + 25 = 50$

$\Rightarrow y^2 + 4y + 4 = 16$

$\Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow (y+6)(y-2) = 0$

$\Rightarrow y = 2, -6$.

The points are (0, 2, 0) and (0, -6, 0).

13. Here A and B are two points whose coordinates are (2, -3, 4) and (8, 0, 10), respectively.

Let P divides the line AB in the ratio k : 1.

The coordinates of P are

$$\left(\frac{8k+2}{k+1}, \frac{ok-3}{k+1}, \frac{10k+4}{k+1}\right)$$

Since its x - coordiante is 4,

$$\frac{8k+2}{k+1}=4$$

$$\Rightarrow 8k+2=4k+4$$

$$\Rightarrow 4k=2$$

$$\Rightarrow k=\frac{1}{2}.$$

The coordinates of P are

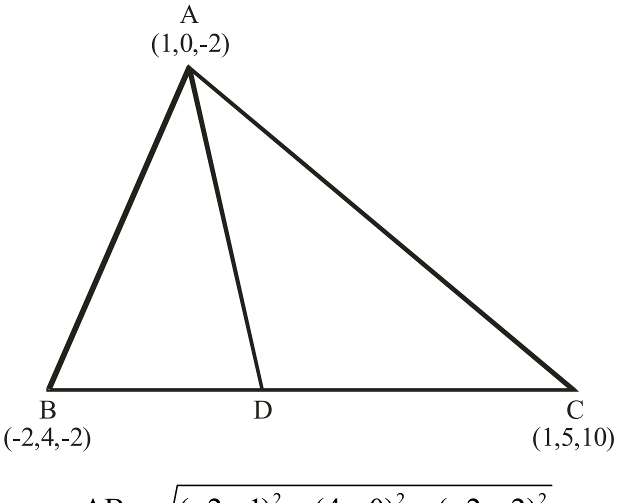
$$\left(\frac{8.\frac{1}{2}+2}{\frac{1}{2}+1}, \frac{-3}{\frac{1}{2}+1}, \frac{10.\frac{1}{2}+4}{\frac{1}{2}+1}\right)$$

$$=(4, -2, 6).$$

14. ABC is a triangle.

The coordinates of A, B and C are (1, 0, -2), (-2, 4, -2) and (1, 5, 10) respectively.

Let AD be the bisector of the angle A.



$$AB=\sqrt{(-2-1)^2+(4-0)^2+(-2+2)^2}$$

$$=\sqrt{9+16}=\sqrt{25}=5$$

$$AC=\sqrt{(1-1)^2+(5-0)^2+(10+2)^2}$$

$$=\sqrt{25+144}=\sqrt{169}=13\cdot$$

Since AD is the bisector of the angle A, we have

$$\frac{BD}{CD}=\frac{AB}{AC}=\frac{5}{13}$$

Thus D divides BC in the ratio 5 : 13.

The coordinates of D are

$$\left[\frac{5.1+13(-2)}{5+13}, \frac{5.5+13.4}{5+13}, \frac{5.10+13.(-2)}{5+13}\right]$$

$$=\left[\frac{5-26}{18}, \frac{25+52}{18}, \frac{50-26}{18}\right]$$

$$=\left(-\frac{21}{18}, \frac{77}{18}, \frac{24}{18}\right).$$

15. Let A and B be two given points whose coordinates are (2, -3, 1) and (3, -4, -5) respectively.

The equation of the given surface is

$$2x+y+z=7 \quad \text{... (1)}$$

Let AB intersect the surface (1) at C and C divides AB in the ratio k : 1.

The coordiantes of C are

$$\left(\frac{3k+2}{k+1}, \frac{-4k-3}{k+1}, \frac{-5k+1}{k+1}\right)$$

Since this point is on the surface (1), we have

$$2\frac{(3k+2)}{k+1}+\frac{-4k-3}{k+1}+\frac{-5k+1}{k+1}=7$$

$$\Rightarrow 6k+4-4k-3-5k+1=7k+7$$

$$\Rightarrow -3k+2=7k+7$$

$$\Rightarrow 10k=-5$$

$$\Rightarrow k=-\frac{5}{10}=-\frac{1}{2}$$

The line AB divides the plane (1) in the ratio -1:2.

GROUP - C

LONG TYPE QUESTIONS

Each questions carries 6 Marks.

7. Sets, Relation & Functions, Trigonometric functions.

1. Show that $A-\bigcup_{i=1}^{n} B_c = \bigcap_{i=1}^{n}(A-B_i)$

2. Show that $A-\bigcap_{i=1}^{n} B_c = \bigcup_{i=1}^{n}(A-B_i)$

3. Prove that $\left(\bigcup_{i=1}^{n} A_i\right)' = \bigcap_{i=1}^{n} A_i$

4. Prove that

   (i) $A-(B\cup C)=(A-B)\cap(A-C)$

   (ii) $A-(B\cap C)=(A-B)\cup(A-C)$

5. In a survey of 500 students in a school, 120 were listed as taking apple juice, 150 as taking orange juice and 100 were listed as taking both apple as well as orange juice. Find now many students we taking neither apple juice nor orange juice.

6. If R={(a, a^3) : a is a prime number less than 10}

   then find (i) R, (ii) dom R, (iii) rng R, (iv) R^{-1}, (v) dom R^{-1} and (vi) rng R^{-1}.

7. If f(x) = cos (log x), then show that

$$f(x^2).f(y^2)-\frac{1}{2}\left[f\left(\frac{x^2}{y^2}\right)+f(x^2y^2)\right]=0.$$

8. If $f(x)=\log_e\left(\frac{1+x}{1-x}\right)$, then show that

$$f\left(\frac{2x}{1+x^2}\right).=2f(x).$$

9. If n be a positive integer and a function f is defined as

$$f(n)=\begin{cases}0 & \text{when } n=1\\ f\left(\left[\frac{n}{2}\right]\right)+1 & \text{when } n>1\end{cases}$$

   then find f(35).

10. If for non zero x,

$$a\ f(x)+b f\left(\frac{1}{x}\right)=\frac{1}{x}-5$$

   where $a\neq b$, then find f(x).

11. Prove that

$$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$$

$$=2\cot^n\frac{A-B}{2}\text{ or zero}$$

   according as n is even or odd.

12. If $\tan\beta=\frac{n^2\sin\alpha\cos\alpha}{1-n^2\sin^2\alpha}$, then

   show that $\tan(\alpha-\beta)=(1-n^2)\tan\alpha$.

13. Find the number of values of x in the interval $[0, 5\pi]$ satisfying the equation

   $3\sin^2x - 6\sin x + 2 = 0$

14. If $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ then

   prove that c = 60.

15. If $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$ are the sides of a triangle, then prove that the measure of the greatest angle is 120^0.

8. Principle of Mathematical Induction, Complex Numbers and Quadratic Equations, Linear Inequalities, Permutation and Combination, Binomal theorem, Sequence and Series.

1. Prove by using principle of induction

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

2. Prove by principle of induction, $n^3 - n$ is divisible by 3 for $n \in N$.

3. If 1, w and w^2 are cube roots of unity then

$(1 - w + w^2)(1 - w^2 + w^4)(1 - w^4 - w^8)$ ...

to 2n factors = 2^{2n}.

4. If $z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1 = 0$ then show that $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$.

5. If $z = x + iy$ then show that

$$|x| + |y| \le \sqrt{2}|z|$$

6. What is the total number of integers with distinct digits that exceed 5500 and do not contain 0, 7 and 9 ?

7. In how many ways can the letters of the word "FAILURE" be arranged so that vowels are not separated.

8. If $n_{p_r} = 1680$ and $n_{c_r} = 70$

then find n are r.

9. A committee of 5 is to be chosen from 6 gentlemen and 4 ladies. In how many ways can the selection be made so as to include atleast one lady ?

10. A bag contains 4 black and 5 while balls out of which 6 balls are drawn arbitrarily. In how many ways can this be done ? Find also the number of ways such that at least 3 black balls can be drawn.

11. Prove that

$$C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n$$

$$= \frac{(2n)!}{(n-r)!(n+r)!}$$

12. If S be the sum, P be the product and R be the sum of reciprocals of n terms of a G. P., then prove that

$$\left(\frac{S}{R}\right)^n = P^2.$$

13. Find the sum of the series

0.7 + 0.77 + 0.777 + 0. 7777 + ... to n terms.

14. Find the value of

$$\frac{1^4}{1!}+\frac{2^4}{2!}+\frac{3^4}{3!}+...$$

15. If α and β are the roots of the equation $ax^2 + bx + c = 0$,

then show that

$$\log_e(a - bx + cx^2) = \log_e a + (\alpha+\beta)x$$

$$-\frac{1}{2}(\alpha^2+\beta^2)x^2 + \frac{1}{3}\left(\alpha^3+\beta^3\right)x^3...$$

9. Straight lines, Conoc sections, Introduction to thre dimensional geometry.

1. If the lines $2x + y + 3 = 0$ and $3x - y - 2 = 0$ are concurrent, then find the value of k.

2. Find the image of the point (1, 2) with respect to the line $x - 3y + 4 = 0$.

3. Show that the area of the triangle formed by the lines $y = m_1n + c_1$ and $y = m_2x + c_2$ and y-axis is $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$.

4. Find the equation of the line parallel to $3x - 4y - 5 = 0$ at a unit distance from it.

5. Two sides of a square lie on the line $x + y = 1$ and $x + y + 2 = 0$. Find its area.

6. Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

7. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

8. Find the equation of the parabola passing through the points (1, 2), (-2, 3) and (2, -1) and axis being parallel to x-axis.

9. Find the centre, eccentricity, foci, vertices, and points of later a recta, length of a latus rectum and the equation of the axes and directrix of the ellipse $16x^2 + 25y^2 - 32x + 50y - 359=0$.

10. Find the equation of the hyperbola whose foci are at $(\pm 6, 0)$ and whose latus rectum is 10.

11. If the origin is the centroid of the triangle ABC with vertices A (2a, 2, 6), B (-4, 3b, 10) and C(8, 14, 2C), find the values of a, b and c.

12. Find the coordinates of a point on y-axis, which are at a distance of $5\sqrt{2}$ from the point (3, -2, 5).

13. A point P with x-coordinate 4 lies on the line segment joining the points A (2, -3, 4) and B(8, 0, 10). Find the coordinates of P.

14. A, B, C are the vertices of a triangle ABC whose coordinator are (1, 0, -2), (-2, 4, -2) and (1 5, 10) respectively. The bisector of the angle BAC meets BC at D. Find the coordinates of D.

15. Find the ratio in which the line segment joining the points (2, -3, 1) and (3, -4, -5) is divided by the lous $2x + y + z = 7$.

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2
1
3
60
50
40
30
20
10